NORTH MAHARASHTRA UNIVERSITY, JALGAON.

Syllabus for F.Y.B. Sc. Mathematics
with effect from June 2015.(Semester system).

SEMESTER –I

MTH-111: Matrices.
MTH-112: Calculus of one Variable.
MTH-113(A): Geometry.
OR
MTH-113(B): Discrete Mathematics.

SEMESTER- II.

MTH-121: Ordinary Differential Equations.
MTH-122: Theory of Numbers and Equations.
MTH-123(A) : Laplace Transforms.
OR
MTH-123(B): Numerical Methods.

SEMESTER-I

MTH - 111: Matrices.

Unit-1. Adjoint and Inverse of a Matrix : Periods-11, Marks-15.

Elementary operations on matrices. Adjoint of a matrix. Inverse of a matrix. Existence & uniqueness theorem of inverse of a matrix. Properties of inverse of a matrix.

Unit-2. Rank of a Matrix : Periods-11, Marks-15.

Elementary matrices. Rank and normal form of a matrix. Reduction of a matrix to its normal form. Rank of product of two matrices.

Unit-3. System of Linear Equations and Eigen values :
Periods-11, Marks-15.

A homogeneous and non-homogeneous system of linear equations. Consistency of system of linear equations. Application of matrices to solve the system of linear equations. Eigen values, Eigen vectors & Characteristic equation of a matrix. Cayley Hamilton theorem (statement only) and its use to find the inverse of a matrix.

Unit-4. Orthogonal matrices and Quadratic forms :
Periods-12, Marks-15.

Orthogonal matrices & Properties of orthogonal matrices. Quadratic forms: Matrix representation, Rank of a quadratic form. Linear transformations. Congruent matrices, Elementary congruent transformations, Diagonal form of a quadratic form, Canonical forms.

References:

1. Matrix and Linear Algebra, by K. B. Datta, Prentice Hall of India Pvt. Ltd. New Delhi,2000.
2. Matrices, by Shantinarayan
3. Matrices, Schaum's outline series

MTH-112: Calculus of one Variable

Unit-1. Limits and Continuity : Periods-11, Marks-15.

Epsilon-delta definition of limit of a function . Basic properties of limits. Indeterminate forms. L-Hospitals rule. Continuous functions. Properties of continuous functions on closed and bounded intervals. Theorems on Boundedness of continuous functions, including Intermediate value theorem. Uniform continuity.

Unit-2. Mean Value Theorems : Periods-11, Marks-15.
Differentiability. Rolle's Theorem. Lagrange's Mean Value Theorem. Cauchy's Mean Value Theorem. Geometrical interpretation and applications.

Unit-3. Successive Differentiation : Periods-11, Marks-15.

The n^{th} derivative of some standard functions: (i) $e^{(ax+b)}$ (ii) $(ax+b)^m$ (iii) x^m (iv)$1/(ax+b)$ (v) $\log(ax+b)$ (vi) $\sin(ax+b)$ (vii) $\cos(ax+b)$ (viii) $e^{ax}\sin(bx+c)$ (ix) $e^{ax}\cos(bx+c)$. Leibnitz's Theorem & Examples.

Unit-4. Taylor's Theorem, Maclaurin's Theorem and Reduction formulae: Periods-12, Marks-15.
Taylor's theorem with Lagranges form of remainder and related examples. Maclaurin' theorem with Lagranges form of remainder and related examples. Reduction formulae i) $\int_0^{\pi/2}\sin^n x dx$ ii) $\int_0^{\pi/2}\cos^n x dx$ iii) $\int_0^{\pi/2}\sin^m x\cos^n x dx$ iv) $\int\frac{\sin nx}{\sin x}dx$.

References:

1. Theory and Problems of Advance calculus, by Murray R. Spiegel, Schaum outline series, Schaum pub. Co. New York.
2. Differential calculus, by Gorakh Prasad, Pothishala Private Ltd. Allahabad.
3. Integral calculus, by Gorakh Prasad, Pothishala Private Ltd. Allahabad.

MTH-113(A): Geometry.

Unit-1. Analytical Plane Geometry : Periods-11, Marks-15.
Change of axes. Translation and Rotation. Invariants. Conic sections. General equation of second degree in two variables and its reduction to standard form.

Unit-2. Sphere : Periods-11, Marks-15.

Equation of sphere in different forms. Plane section of sphere. Tangent line and Tangent plane to sphere. Condition of tangency and point of contact. Interpretation of $S + \lambda S' = 0$ and $S + \lambda U = 0$ with usual notations.

Unit-3. Cone : Periods-11, Marks-15.

Equation of cone with vertex at origin. Equation of cone with vertex at (α, β, γ). Right circular cone. Enveloping cone of sphere. Tangent line and tangent plane to the cone.

Unit-4. Cylinder : Periods-12, Marks-15.

Definition and Equation of cylinder. Right circular cylinder. Enveloping cylinder.

References:

1. The elements of co-ordinate geometry, by S. L. Loney, MacMillan and company, London.
2. Text book on co-ordinate geometry, by Gorakh Prasad and H.C. Gupta, Pothishala pvt. Ltd. Allhabad.
3. Analytical Solid geometry, by Shantinarayan, S. Chand & Co.

MTH-113 (B) Discrete Mathematics

Unit-1. Graphs : Periods-11, Marks-15.

Definition, Simple graph, Multigraph. Hand shaking lemma. Types of graphs. Operations on graphs. Subgraphs. Isomorphism of graphs.

Unit-2. Connected Graphs : Periods-11, Marks-15.

Walk, path , cycles (circuits), Connected and disconnected graphs, Cut vertices and connectivity. Eulerian graph. Konigsberg seven bridge problem. Hamiltonian graph. Traveling salesman problem.

Unit-3. Planar and Dual Graphs : Periods-11, Marks-15.

Planar graphs. Euler's formula for planar graphs. Kuratowskis two graphs. Geometrical dual. Coloring of the graphs. Directed graphs. Types of Digraphs.

Unit-4. Trees : Periods-12, Marks-15.

Definition and properties of trees. Distance and centers in a tree. Rooted and binary trees. Spanning tree. Minimal spanning tree.

References:

1. Graph theory and Boolean Algebra by J. N. Salunkhe, Sonu Nilu Publication Nagpur.
2. Graph Theory with applications to Engineering and computer science by Narsingh Deo. Prentice Hall of India Pvt. Ltd.
3. Discreate Mathematics by Seymour Lipschutz, Schaum's outline series.

SEMESTER-II

MTH-121: Ordinary Differential Equations.

Unit-1. Differential equations of first order and first degree :
Periods-11, Marks-15.

Partial derivatives of first order & second orders and Examples. Exact differential equations. Condition for exactness. Integrating factor. Rules for finding integrating factors. Linear differential equations. Bernoulli's Equation. Equation reducible to linear form.

Unit-2. Differential equations of first order and higher degree :
Periods-11, Marks-15.

Differential equations of first order and higher degree. (a) solvable for p. (b) solvable for y. (c) solvable for x. (d) Clairaut's form.

Unit-3. Linear differential equations of second and higher order :
Periods-11, Marks-15

Linear differential equations with constant coefficients. Complementary functions. Particular integrals of $f(D)y = X$, where $X = e^{ax}$, $\cos(ax)$, $\sin(ax)$, x^n, $e^{ax}V$, xv with usual notations.

Unit-4. Homogeneous linear differential equations :
Periods-12, Marks-15.

Homogeneous linear differential equations (Cauchy's differential equations). Equations reducible to homogeneous linear differential equations (Legendre's equations) with examples.

References:

1. Introductory course in Differential Equations, by D. A. Murray, Orient Congman (India) 1967.
2. Differential Equations , by G. F. Simmons,Tata McGraw Hill, 1972.
3. Differential Equations by J. N. Salunkhe, Sonu Nilu publication.
4. Differential Equation, by Sharma, Vashistha, Meerut publication.

MTH-122: Theory of Numbers and Equations

Unit-1. Divisibility of Integers : Periods-11, Marks-15.

Natural numbers. Well ordering principal (statement only). Principle of Mathematical Induction. Divisibility of integers and theorems. Division algorithm. GCD and LCM. Euclidean algorithm. Unique factorization theorem.

Unit-2. Congruence classes : Periods-11, Marks-15.

Partition of a set. Equivalence relations. Equivalence classes. Congruence relation (modulo n) and theorems. Properties of residue classes. Composition tables. Fermat's theorem. Euler's function ϕ. Euler's theorem.

Unit-3. Theory of Equations-I : Periods-11, Marks-15.

Relation between roots and coefficient of general polynomial equation in one variable . Relation between roots and coefficient of quadratic, cubic and biquadratic equations. Symmetric functions of roots.

Unit-4. Theory of Equations –II : Periods-12, Marks-15.

Transformation of equations. Descarte's rule of signs. Cardon's method of solving cubic equations. Biquadratic equations. Descarte's method of solving biquadratic equations.

References:

1. Elementary Number Theory, by David M. Button, W. C. Brown publishers, Dubuquo lowa 1989.
2. Higher Algebra, by H. S. Hall and S. R. Knight, H. M. Publications 1994.
3. Matrix and Linear Algebra, by K. B. Datta, Prentice Hall of India Pvt. Ltd. New Delhi, 2000.

MTH – 123 (A) : Laplace transforms

UNIT-1. Laplace Transforms : Periods-11, Marks-15.
Definition and existence of Laplace transform. Laplace transforms of elementary functions and validity. $f(t) = 1$, e^{kt}, cosh(kt), sinh(kt), cos(kt), sin(kt), t^n ($t > 0$, $n > 1$). Idea of sectionally continuous functions and of exponential order. Properties of Laplace transform. Laplace transforms of derivatives. Laplace transforms of real integrals. Multiplication by t^n. Division by t. Definition and simple properties of Beta and Gamma functions (without proof).

UNIT-2. Inverse Laplace Transform: Periods-11, Marks-15.
Definition and use of table. Properties of inverse Laplace Transforms : Linearity, first and second translation, Change of scale. Inverse Laplace transforms of derivatives. Inverse Laplace transforms of integrals. Multiplications by s . Division by s.

UNIT-3. Convolution Theorem : Periods-11, Marks-15.
Laplace transforms of periodic functions. convolution theorem (without proof). Evaluation of inverse Laplace transform by convolution theorem. Use of partial fractions.

UNIT-4. Applications to Differential Equations :
Periods-12, Marks-15.
Solution of linear differential equation with constant coefficients by using Laplace transforms. Laplace transforms of Heavisides unit step functions. Laplace transforms of Dirac-Delta functions.

Reference books -

1. Theory and problems of Laplace transforms, by Murry R. Spiegel, Schaum’s Outline Series in Mathematics
2. Integral transforms, by A. R. Vasishtha and R. K. Gupta.

MTH – 123 (B): Numerical Methods

UNIT-1 : Solution of Algebraic and Transcendental Equations.
Period : 11, Marks : 15

Errors and their computation. Absolute, relative and percentage errors. The bisection method. The iteration method. The method of false position. Newton-raphson method.

UNIT-2 : Interpolation **Period : 11, Marks : 15**

Finite differences : Forward differences, backward differences, central difference. Symbolic relations and separation of symbols. Gauss's forward central difference formula. Gauss's backward central difference formula. Interpolations with unevenly spaced points. Lagrange's interpolation formula. Inverse Lagrange's Formula

UNIT-3 : Curve Fitting **Period : 11, Marks : 15**

Least squares curve fitting procedures. Fitting of straight line. Non-linear curve fitting: power function $y = ax^c$. Fitting of polynomial of degree two $y = a + bx + cx^2$. Fitting of exponential function $y = ae^{bx}$.

UNIT-4: Numerical Solutions of Ordinary Differential Equations
Period : 12, Marks : 15

Solution by Taylor's series. Euler's method. Modified Euler's method. Runge kutta methods: Runge kutta second and fourth order formulae.

Reference books -

1. Introductory methods of numerical analysis: by S.S.Sastry (Third Edition)
2. Introduction to numerical analysis (second edition) by C.E.Froberg Addison-Wesley, 1979
3. Numerical Methods by V.N. Vedamurty and N.Ch.S.H. Iyehgar,Vikas Publication India
4. Numerical methods for scientific and engineering computation: By M.K.Jain, S.R. K. Iyenger, R.K. Jain. New Age International (P) ltd, 1999.

The equivalences for Old courses of F. Y. B. Sc. Mathematics is given as follows:

Sem	Old Course (June 2012)	New equivalent course (June 2015)
I	MTH : 111 – Theory of Matrices	MTH : 111 – Matrices
	MTH : 112 – Calculus	MTH : 112 – Calculus of one variable
	MTH : 113(A) – Co-ordinate Geometry	MTH : 113(A) – Geometry
	MTH : 113(B) – Graph Theory	MTH : 113(B) – Discrete Mathematics
II	MTH : 121 – Differential Equations	MTH : 121 – Ordinary Differential Equations
	MTH : 122 – Algebra	MTH : 122 – Theory of Numbers and Equations
	MTH : 123(A) – Laplace Transforms	MTH : 123(A) – Laplace Transforms
	MTH : 123(B) – Computational Mathematics	MTH : 123(B) – Numerical Methods

Opportunities for mathematicians (Under Gradate)

Between one third and one half of all jobs requiring graduates are open to students of any discipline. Of course, mathematicians are eligible for these jobs. In addition, there are careers for which a degree in mathematics is either essential or a strong advantage. These fall into a number of general areas:

1. Scientific research, design and development
2. Management services and computing
3. Financial work
4. Statistical work
5. Teaching
6. Postgraduate study

Finally, a degree in mathematics does not train you for a specific job. Rather it gives you a range of skills which enable you to enter any of a wide range of careers. It is therefore a versatile qualification. By taking a mathematics degree, you are able to make your career choice when you are 21 rather than when you are 18. Your aspirations may well have changed during the intervening years. Moreover, you will have a clearer understanding of the work you would be doing and you will have been able to talk with representatives of the companies who will wish to employ you. Three years at a university/College will broaden your horizons in many ways. There is no need to narrow your career horizon while you are still at school unless you so wish.

NORTH MAHARASHTRA UNIVERSITY JALGAON.

Syllabus for S.Y.B.Sc. (Mathematics)

With effect from June 2016. (Semester system).

The pattern of examination of theory and practical papers is semester system. Each paper is of 100 marks (60 marks external and 40 marks internal). and practical course is of 100 marks (60 marks external and 40 marks internal). The examination will be conducted at the end of each semester.

STRUCTURE OF COURSES

SEMESTER –I

MTH-231 : Calculus of Several Variables

MTH-232 (A) : Algebra
OR
MTH-232 (B) : Theory of Groups

MTH- 233 : Practical Course based on MTH-231, MTH-232

SEMESTER- II.

MTH-241 : Complex Variables

MTH-242 (A) : Differential Equations
OR
MTH-242(B) : Differential and Difference Equations

MTH-243 : Practical Course based on MTH-241, MTH-242

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SEMESTER – I

MTH -231: Calculus of Several Variables

Unit- 1 : Functions of Two and Three variables **Periods -15 ,Marks – 15**

1.1 Limits and Continuity
1.2 Partial Derivatives and Jacobians .
1.3 Higher order Partial Derivatives
1.4 Differentiability and Differentials
1.5 Necessary and sufficient conditions for Differentiability
1.6 Schwarz's Theorem
1.7 Young's Theorem.

Unit- 2 : Composite Functions and Mean Value Theorem **Periods – 15, Marks – 15**

2.1 Composite functions. Chain Rule.
2.2 Homogeneous functions.
2.3 Euler's Theorem on Homogeneous Functions.
2.4 Mean Value Theorem for Function of Two Variables.

Unit -3 : Taylor's Theorem and Extreme Values **Periods – 15, Marks – 15**

3.1 Taylor's Theorem for a function of two variables.
3.2 Maclaurin's Theorem for a function of two variables.
3.3 Absolute and Relative Maxima and Minima.
3.4 Necessary Condition for extrema.
3.5 Critical Point, Saddle Point.
3.6 Sufficient Condition for extrema.
3.7 Lagrange's Method of undetermined multipliers.

Unit -4 : Double and Triple Integrals **Periods – 15, Marks – 15**

4.1 Curve Tracing
4.2 Double integrals by using Cartesian and Polar coordinates.
4.3 Change of Order of Integration.
4.4 Area by Double Integral.
4.5 Evaluation of Triple Integral as Repeated Integral.
4.6 Volume by Triple Integral.

Reference Books –

1. Mathematical Analysis: S.C. Malik and Savita Arora. Wiley Eastern Ltd, New Delhi.
2. Calculus of Several Variables: Schaum's outline Series.
3. Mathematical Analysis: T.M.Apostol. Narosa Publishing House, New Delhi, 1985
4. A Course of Mathematical Analysis: Shanti Narayan, S. Chand and Company, New Delhi.

MTH -232(A): Algebra

Unit-1 : Groups **Periods-15, Marks-15.**

1.1 Definition of a group.

1.2 Simple properties of group.

1.3 Abelian group.

1.4 Finite and infinite groups.

1.5 Order of a group.

1.6 Order of an element and its properties.

Unit-2 : Subgroups **Periods-15, Marks-15.**

2.1 Definition of subgroup, criteria for a subset to be a subgroup.

2.2 Cyclic groups.

2.3 Dihedral group (Definition and Examples only)

2.4 Coset decomposition.

2.5 Lagrange's theorem for finite group.

2.6 Euler's theorem and Fermat's theorem.

Unit -3 : Homomorphism and Isomorphism of Groups **Periods-15, Marks-15.**

3.1 Definition of Group Homomorphism and its Properties .

3.2 Kernel of Homomorphism and Properties.

3.3 Definition of Isomorphism and Automorphism of Groups.

3.4 Properties of Isomorphism of Groups.

Unit-4 : Rings **Periods-15, Marks-15.**

4.1 Definition and Simple Properties of a Ring.

4.2 Commutative Ring, Ring with unity, Boolean Ring.

4.3 Ring with zero divisors and without zero Divisors.

4.4 Integral Domain, Division Ring and Field. Simple Properties.

Reference Books –

1. Topics in Algebra: I. N. Herstein (John Wiley and Sons).
2. A first Course in Abstract Algebra: J. B. Fraleigh (Pearson).
3. University Algebra: N. S. Gopalakrishnan (New age international publishers).
4. A course in Abstract Algebra: Vijay K. Khanna and S.K.Bhambri (Vikas Publishing House Pvt, Ltd. Noida).

MTH -232(B): Theory of Groups

Unit-1 : Groups **Periods-15, Marks-15.**

1.1 Definition of a group.
1.2 Simple properties of group.
1.3 Abelian group.
1.4 Finite and infinite groups.
1.5 Order of a group.
1.6 Order of an element and its properties.

Unit-2 : Subgroups **Periods-15, Marks-15.**

2.1 Definition of subgroup, criteria for a subset to be a subgroup.
2.2 Cyclic groups.
2.3 Dihedral group (Definition and Examples only)
2.4 Coset decomposition.
2.5 Lagrange's theorem for finite group.
2.6 Euler's theorem and Fermat's theorem.

Unit -3 : Homomorphism and Isomorphism of Groups **Periods-15, Marks-15.**

3.1 Definition of group homomorphism and its properties .
3.2 Kernel of homomorphism and properties.
3.3 Definition of isomorphism and automorphism of groups.
3.4 Properties of isomorphism of groups.

Unit-4 : Group Codes **Periods -15, Marks 15.**

4.1 Message, Word,(m,n)- Encoding Function, Code Words.
4.2 Detection of k or fewer errors, Weight, Parity Check Code
4.3 Hamming Distance, Properties of the Distance Function, Minimum Distance of an encoding function.
4.4 Group Codes.
4.5 (n, m)- Decoding function, Maximum Likelihood Decoding Function.
4.6 Decoding procedure for a Group Code given by a Parity Check Matrix.

Reference Books –

1. Discrete Mathematical Structures: Bernard Kolman, Robert C. Busby and Ross (Prentice Hall of India New Delhi, Eastern Economy Edition).
2. Topics in Algebra: I.N. Herstein.
3. A first Course in Abstract Algebra: J. B. Fraleigh (Pearson).
4. University Algebra: N. S. Gopalakrishnan (New age international publishers).
5. A course in Abstract Algebra: Vijay K. Khanna and S.K.Bhambri (Vikas Publishing House Pvt, Ltd. Noida).

MTH- 233 : Practical Course based on MTH-231, MTH-232

Practical – 1 : Functions of Two or Three Variables

1. Evaluate limit if it exists for following functions.

   i) $\lim_{(x,y)\to(0\ ,\ 0)} \frac{\sin(x^2+y^2)}{x+y}$, $x + y \neq 0$

   ii) $\lim_{(x,y)\to(0\ ,\ 0)} \frac{x^3y^3}{x^2+y^2}$, $x^2 + y^2 \neq 0.$

2. Let $f(x,y) = \frac{x^2y^2}{x^4+y^4-x^2y^2}$, $(x,y) \neq (0\ ,\ 0)$, verify that both the repeated limits exist and are equal but simultaneous limit does not exist.

3. i) Discuss the continuity of $f(x\ ,\ y) = \begin{cases} \frac{2xy^2}{x^3+y^3}\ , & \text{if } (x\ ,\ y) \neq (0\ ,\ 0) \\ 0\ , & \text{if } (x\ ,\ y) = (0\ ,\ 0) \end{cases}$ at origin.

   ii) Show that the function $f(x\ ,\ y) = \begin{cases} \frac{x^3-y^3}{x^2+y^2}\ , & \text{if } (x\ ,\ y) \neq 0 \\ 0\ , & \text{if } (x\ ,\ y) = (0\ ,\ 0) \end{cases}$

   is continuous at origin.

4. If $u = \log(\tan x + \tan y + \tan z)$, then prove that $\sin 2x \frac{\partial u}{\partial x} + \sin 2y \frac{\partial u}{\partial y} + \sin 2z \frac{\partial u}{\partial z} = 2.$

5. If $f(x\ ,\ y) = \begin{cases} \frac{x^3y}{x^2+y^2}\ , & \text{if } x^2 + y^2 \neq 0 \\ 0\ , & \text{if } (x\ ,\ y) = (0\ ,\ 0) \end{cases}$, then show that $f_{xy}(0\ ,\ 0) \neq f_{yx}(0\ ,0)$.

6. Show that the function $f(x,y) = \sqrt{|xy|}$ is continuous at (0 , 0) but not differentiable at (0 , 0).

7. Find approximately the value of $[(3.8)^2 + 2(2.1)^3]^{\frac{1}{5}}$.

8. Find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$, where $u = x + y + z$, $v = x + y$, $w = x - y - z$.

9. If $x = r\cos\theta$, $y = r\sin\theta$, Verify that $J.J' = 1$.

Practical - 2 : Composite functions and Mean Value Theorem

1. If $z = e^{xy^2}$, $x = t\cos t$, $y = t\sin t$, find $\frac{dz}{dt}$ at $t = \frac{\pi}{2}$.
2. If $w = f(x-y,\ y-z,\ z-x)$, find the value of $\frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} + \frac{\partial w}{\partial z}$.
3. If z is function of x and y and if $x = e^u + e^{-v}$, $y = e^{-u} - e^v$, then prove that $\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y}$.
4. If $z = f(x, y)$, where $u = 2x - 3y$, $v = x + 2y$, then prove that $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 3\frac{\partial z}{\partial v} - \frac{\partial z}{\partial u}$.
5. If $u = f(e^{y-z},\ e^{z-x},\ e^{x-y})$, then prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$.
6. If $u = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$, then show that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\,\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \sin(1 - 4\sin^2 u)$.
7. If $u = x\phi\left(\frac{y}{x}\right) + \psi\left(\frac{y}{x}\right)$, then prove that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\,\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 0$.
   Given that ϕ and ψ are at least twice differentiable.
8. If $f(x, y) = x^3 - xy^2$, show that θ used in the Mean value theorem applied to the points $(2, 1)$ and $(4, 1)$ satisfies the quadratic equation $3\theta^2 + 6\theta - 4 = 0$.
9. Let $f(x, y) = x^2y + 2xy^2$. Find the quadratic equation in θ by applying the Mean Value Theorem to line segment joining the points (1 , 2) to (3 , 3).

Practical – 3 : Taylor's Theorem and Extreme Values

1. Show that expansion of $\sin(xy)$ in powers of (x - 1) and (y - $\frac{\pi}{2}$) up to and including second term is
   $1 - \frac{\pi^2}{8}(x-1)^2 - \frac{\pi}{2}(x-1)(y-\frac{\pi}{2}) - \frac{1}{2}(y-\frac{\pi}{2})^2$.
2. Show that , for $0 < \theta < 1$
   $\sin x\ \sin y = xy - \frac{1}{6}[(x^3 + 3xy^2)(\cos\theta x \sin\theta y) + (y^3 + 3x^2y)\sin\theta x \cos\theta y]$.
3. Prove that $\sin(x+y) = (x+y) - \frac{(x+y)^3}{3!} + \cdots$
4. Expand $e^{2x}\cos y$ as Taylor's series about $(0, 0)$ up to first three terms.
5. Find the points (x , y) where the function u = xy (a - x - y) is maximum or minimum. What is the maximum value of the function?
6. Find the least value of the function $f(x, y) = xy + \frac{50}{x} + \frac{20}{y}$
7. Determine the minimum distance from the origin to the plane $3x + 2y + z - 12 = 0$.
8. Divide 24 in to three positive numbers such that their product is maximum.
9. Find the dimensions of a rectangular box open at the top whose volume is 108 cubic meters and its surface area is minimum.

Practical – 4 : Double and Triple integrals

1. Find the area bounded by the parabolas $y^2 = 2x$ and $x^2 = 2y$.
2. Evaluate $\int_0^1 \int_0^{x^2} e^{\frac{y}{x}}\, dxdy$
3. Using triple integration, find the volume of the sphere of radius 5
4. Evaluate $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z}\, dxdydz$
5. Change the order of integration $\int_0^1 \int_{x^2}^{2-x} f(x,y)dxdy$

6. Change the order of integration $\int_0^3 \int_1^{\sqrt{4-y}} f(x,y)\, dxdy$
7. Evaluate $\int_{x=0}^1 \int_{y=0}^2 \int_{z=1}^2 x^2 yzdzdydx$

8. Evaluate $\int_{y=0}^3 \int_{x=0}^2 \int_{z=0}^1 (x+y+z)dzdxdy$

9. Find the volume bounded by the cylinder $x^2 + y^2 = 4$ and the planes $y + z = 3,\ z = 0$

Practical - 5 : Groups

1. Let $\mathbb{Q}^+$ denote the set of all positive rational numbers and for any a, b $\in \mathbb{Q}^+$ define $a*b = \frac{ab}{2}$. Show that $(\mathbb{Q}^+, *)$ is an abelian group.

2. Let G = { (a , b) : a , b$\in \mathbb{R}$, a $\neq$ 0}. Show that (G , ʘ) is a non- abelian group, where (a , b) ʘ (c , d) = (ac , ad + b).
3. Show that the set G of all 2x2 matrices of the form $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ over $\mathbb{R}$ where ad – bc ≠ 0 forms a non abelian group under matrix multiplication.

4. Let G be a group in which $(ab)^n = a^n b^n$ for three consecutive integers n and for any a, b $\in$ G. Show that G is abelian.

5. Let G = $\left\{\begin{bmatrix} a & a \\ a & a \end{bmatrix} : a \in \mathbb{Q}^+\right\}$. Prove that G is an abelian group with respect to multiplication of matrices.
6. (a) In the group $(\mathbb{Z}_8\,,\ +_8)$, find (i) $\bar{3}^2$ ii) $\bar{3}^{-2}$ iii) $o(\bar{5})$ iv) $o(\bar{7})$
   (b) In the group $(\mathbb{Z}'_{11}\,,\ \times_{11})$, find (i) $\bar{4}^3$ ii) $\bar{4}^{-3}$ iii) $o(\bar{9})$ iv) $o(\bar{7})$
7. If in a group G, the elements a and b commute, then prove that
   (i) a^{-1} and b^{-1} commute (ii) a and b^{-1} commute (iii) a^{-1} and b commute

Practical - 6: Subgroups

1. If H is a subgroup of a group G and $x \in G$, then show that $xHx^{-1} = \{ xhx^{-1} : h \in H\}$ is a subgroup of G.
2. Show that is $(\mathbb{Z}'_7 , \times_7)$ a cyclic group. Find all its generators, all its proper subgroups and order of every element.

3.(a) In a commutative group $(G , *)$, define $H = \{a \in G : a^k = e \text{ for some } k \in \mathbb{N} \}$. Determine whether $(H , *)$ is a subgroup of $(G , *)$.

(b) Let $H = \{ \bar{0} , \bar{2} , \bar{4} , \bar{6} \}$ be a subgroup of a group $G = (\mathbb{Z}_8 , +_8)$. Find all right(left) cosets of H in G.

4.Let A and B be two subgroups of a finite group G whose orders are relatively prime. Show that $A \cap B = \{e\}$.

5. (a) Show that every proper subgroup of a group of order 55 is cyclic.
(b) Find the remainder obtained when 15^{27} is divided by 8.
(c) Find the remainder when 41^{75} is divided by 3.

6. If G is a group of order 10, then show that it must have a subgroup of order 5.

Practical - 7 : Homomorphism and Isomorphism of Groups

1.Let $G = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} : a , b , c , d \in \mathbb{R}, \ ad - bc \neq 0 \right\}$, the group of all nonsingular matrices of order 2 over $\mathbb{R}$ under matrix multiplication and let $\mathbb{R}^* = \mathbb{R} - \{0\}$, the group of nonzero real numbers under multiplication. Define $f : G \to \mathbb{R}^*$ by $f(A) = |A|$, for all $A \in G$. Show that f is an onto group homomorphism and find its kernel.

2. If $G = \{1, -1, i, -i\}$ is a group under multiplication and $\bar{G} = \{ \bar{2} , \bar{4} , \bar{6} , \bar{8}\}$ is a group under multiplication modulo 10, then show that G and $\bar{G}$ are isomorphic.

3. (a) Let G be a group and $a \in G$. Show that $f_a : G \to G$ defined by $f_a(x) = axa^{-1}$, for all $x \in G$ is an automorphism.

(b) Show that the groups $G = \{1, -1, i, -i\}$ under usual multiplication and $\mathbb{Z}'_8 = \{ \bar{1} , \bar{3} , \bar{5} , \bar{7}\}$ under multiplication modulo 8 are not isomorphic.

4.Let G be a group and $f : G \to G$ be a map defined by $f(x) = x^{-1}$, for all $x \in G$. prove that
  a) if G is abelian, then f is an isomorphism.
  b) If f is a group homomorphism, then G is abelian.

5.Show that the set of all automorphisms of a group G forms a group under composition of mappings.

6.Let f and g be group homomorphisms from $G \to G$. Show that $H=\{x \in G : f(x) = g(x)\}$ is a subgroup of G.

Practical - 8(A) : Rings

1.(a) Show that $\mathbb{Z}_7 = \{ \bar{0} , \bar{1} , \bar{2} , \bar{3} , \bar{4} , \bar{5} , \bar{6} \}$ forms a ring under addition and multiplication modulo 7.
(b) In the ring $(\mathbb{Z}_{10} , +_{10} , \times_{10})$, find all divisors of zero.

2. Show that $\mathbb{Z}[i] = \{a + ib : a , b \in \mathbb{Z} \}$, the set of Gaussian integers, forms an integral domain under usual addition and multiplication of complex numbers.

3. Show that $R = \{a + b\sqrt{2} : a , b \in \mathbb{Z}\}$ is an integral domain under usual addition and multiplication.

4. In the ring $(\mathbb{Z}_7 , +_7 , \times_7)$, find
   (i) $-(\bar{4} \times_7 \bar{6})$ (ii) $\bar{3} \times_7 (\overline{-6})$ (iii) $(\overline{-5}) \times_7 (\overline{-5})$
   (iv) Units in $\mathbb{Z}_7$ (v) additive inverse of $\bar{6}$, (vi) zero divisors.
   Is $\mathbb{Z}_7$ a field or an integral domain? Justify.

5. Let $\mathbb{R}$ be the set of all real numbers. Show that $\mathbb{R} \times \mathbb{R}$ forms a field under addition and multiplication defined by $(a , b) + (c , d) = (a+c , b+d)$ & $(a , b) . (c , d) = (ac-bd , ad+bc)$.

Practical - 8 (B) : Group Codes

1. Consider the (3,8) encoding function $e : B^3 \to B^8$ defined by
   e(000) = 00000000, e(001) = 10111000, e(010) = 00101101, e(011) = 10010101,
   e(100) = 10100100, e(101) = 10001001, e(110) = 00011100, e(111) = 00110001
   (a) Find the minimum distance of e. (b) How many errors will e detect ?

2. Show that the (3,6) encoding function $e : B^3 \to B^6$ defined by
   e(000) = 000000 , e(001) = 001100, e(010) = 010011, e(011) = 011111,
   e(100) = 100101, e(101) = 101001, e(110) = 110110, e(111) = 111010
   is a group code. Also find the minimum distance of e.

3.Compute: (a) $\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \oplus \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} * \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

4.Let $H = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ be a parity check matrix. Determine the (2 , 5) group code $e_H : B^2 \to B^5$.

5.Consider the parity check matrix : $H = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Decode the following words relative to a maximum likelihood decoding function associated with e_H :
a) 10100 b) 01101 c) 11011

6.Consider the parity check matrix : $H = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Determine the coset leaders for $N = e_H(B^3)$. Also compute the Syndrome for each coset leader and decode the code 001110 relative to maximum likelihood decoding function.

7.Let the (9,3) decoding function $d : B^9 \to B^3$ be defined by $d(y) = Z_1Z_2Z_3$, where

Z_i = 1, if $\{y_i, y_{i+3}, y_{i+6}\}$ has at least two 1's
= 0, if $\{y_i, y_{i+3}, y_{i+6}\}$ has less than two 1's, i = 1, 2, 3.

If $y \in B^9$, then determine d(y), where (i) y = 101111101 (ii) y = 100111100

SEMESTER – II
MTH -241 : Complex Variables

Unit-1 : Complex numbers **Period-15, Marks-15**

1.1 Complex numbers, modulus and amplitude, polar form
1.2 Triangle inequality and Argand's diagram
1.3 De Moivre's theorem for rational indices and applications
1.4 n^{th} roots of a complex number
1.5 Elementary functions (1) Trigonometric functions of a complex variable.
(2) Hyperbolic functions of a complex variable.

Unit-2: Functions of complex variables **Period-15, Marks-15**

2.1 Limits, Continuity, Derivative.
2.2 Analytic functions, Necessary and sufficient condition for analytic function.
2.3 Cauchy Riemann equations.
2.4 Laplace equations and Harmonic functions
2.5 Construction of analytic functions

Unit-3 : Complex integrations **Period-15, Marks-15**

3.1 Line integral and theorems on it.
3.2 Statement and verification of Cauchy-Gaursat's Theorem.
3.3 Cauchy's integral formulae for f(a), f'(a) and $f^n(a)$
3.4 Taylor's and Laurent's series.

Unit-4 : Calculus of Residues **Period-15, Marks-15**

4.1 Zeros and poles of a function.
4.2 Residue of a function
4.3 Cauchy's residue theorem
4.4 Evaluation of integrals by using Cauchy's residue theorem
4.5 Contour integrations of the type $\int_0^{2\pi} f(cos\theta,\, \sin\theta)d\theta \;\; and \;\; \int_{-\infty}^{\infty} f(x)dx$

Reference books –

1. Complex Variables and Applications ; R.V.Churchill and J.W. Brown.(McGraw-Hill)
2. Theory of Functions of Complex Variables : Shanti Narayan, S. Chand and Company, New Delhi.
3. Complex variables : Schaum's Outline Series.

MTH-242(A): Differential Equations

Unit-1 : Theory of ordinary differential equations **Period-15, Marks-10**

1.1 Lipschitz condition
1.2 Existence and uniqueness theorem
1.3 Linearly dependent and independent solutions
1.4 Wronskian definition
1.5 Linear combination of solutions
1.6 Theorems on i) Linear combination of solutions
ii) Linearly independent solutions
iii) Wronskian is zero
iv) Wronskian is non zero
1.7 Method of variation of parameters for second order L.D.E.

Unit-2 : Simultaneous Differential Equations **Period-15, Marks-10**

2.1 Simultaneous linear differential equations of first order
2.2 Simultaneous D.E. of the form $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$.
2.3 Rule I: Method of combinations
2.4 Rule II: Method of multipliers
2.5 Rule III: Properties of ratios
2.6 Rule IV: Miscellaneous

Unit-3 : Total Differential or Pfaffian Differential Equations **Period-15, Marks-10**

3.1 Pfaffian differential equations
3.2 Necessary and sufficient condition for integrability
3.3 Conditions for exactness
3.4 Method of solution by inspection
3.5 Solution of homogenous equation
3.6 Use of auxiliary equations

Unit-4 : Beta and Gamma Functions **Period-15, Marks-10**

4.1 Introduction
4.2 Euler’s Integrals: Beta and Gamma functions
4.3 Properties of Gamma function
4.4 Transformation of Gamma function
4.5 Properties of Beta function
4.6 Transformation of Beta function
4.7 Duplication formula

Reference Book-

1. Ordinary and Partial Differential Equation by M. D. Rai Singhania, S. Chand & Co.
2 . Mathematical Analysis by S.C. Malik and Savita Arora.

MTH-242(B): Differential and Difference Equations

Unit-1 : Theory of ordinary differential equations **Period-15, Marks-10**

1.1 Lipschitz condition
1.2 Existence and uniqueness theorem
1.3 Linearly dependent and independent solutions
1.4 Wronskian definition
1.5 Linear combination of solutions
1.6 Theorems on i) Linear combination of solutions
ii) Linearly independent solutions
iii) Wronskian is zero
iv) Wronskian is non zero
1.7 Method of variation of parameters for second order L.D.E.

Unit-2 : Simultaneous Differential Equations **Period-15, Marks-10**

2.1 Simultaneous linear differential equations of first order
2.2 Simultaneous D.E. of the form $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$.
2.3 Rule I: Method of combinations
2.4 Rule II: Method of multipliers
2.5 Rule III: Properties of ratios
2.6 Rule IV: Miscellaneous

Unit-3 : Total Differential or Pfaffian Differential Equations **Period-15, Marks-10**

3.1 Pfaffian differential equations
3.2 Necessary and sufficient condition for integrability
3.3 Conditions for exactness
3.4 Method of solution by inspection
3.5 Solution of homogenous equation
3.6 Use of auxiliary equations

Unit-4 : Difference Equations **Period-15, Marks-10**

4.1 Introduction, Order of difference equation, degree of difference equations
4.2 Solution to difference equation and formation of difference equations
4.3 Linear difference equations ,Linear homogeneous difference equations with constant coefficients
4.4 Non homogenous linear difference equation with constant coefficients

Reference Book-

1. Ordinary and Partial Differential Equation by M. D. Rai Singhania, S. Chand & Co.
2. Numerical Methods by Dr.V. N. Vedamurthy and Dr. N. Ch. S. N. Iyengar

MTH- 243 : Practical Course based on MTH-241, MTH-242

Practical - 1 : Complex numbers

1. Determine the region in the Z-plane represented by $|z-3|+|z+3|=10$.

2. $a)$ Using De Moivre's theorem prove

   $i)$ $\cos 7\theta = \cos^7\theta - 21cos^5\theta \cdot \sin^2\theta + 35\,cos^2\theta \cdot \sin^4\theta - 7\cos\theta \cdot cos^6\theta$

   $ii)$ $\sin 7\theta = 7\cos^6\theta . sin\theta - 35\cos^4\theta . \sin^3\theta + 21\cos^2\theta . \sin^5\theta - \sin^7\theta$

   $b)$ Using De Moivre's theorem express $cos^6\theta$ in terms of the cosines of multiple angle.

3. $a)$ Find all values of $(1-i)^{\frac{2}{5}}$

   $b)$ Solve the equation $x^8 - x^4 + 1 = 0$.

4. Find the real and imaginary parts of the following

   $a)$ $\cos(x+iy)$, $b)$ $\cosh(x+iy)$

5. If $\sin(a+ib) = x+iy, then\ show\ that\ (a)\ \frac{x^2}{\cosh^2 b} + \frac{y^2}{\sinh^2 b} = 1 \qquad (b)\ \frac{x^2}{\sin^2 a} - \frac{y^2}{\cos^2 a} = 1$

Practical - 2 : Functions of complex variable

1. $a)$ Evaluate $\lim\limits_{z\to i} \frac{3z^4-2z^3+8z^2-2z+5}{z-i}$ $\qquad b)$ Evaluate $\lim\limits_{z\to e^{i\pi/3}} \frac{(z-e^{\frac{i\pi}{3}})z}{z^3+1}$

2. Find $f(1+i)$, if f is continuous at $z=1+i$, where $f(z) = \frac{z^4+4}{z-(1+i)}$, $\quad z \neq 1+i$

3. a) Find an analytic function $f(z) = u+iv\ ,if\ \ v = e^{-y}sinx\ \ and\ f(0) = 1$.

   b) Find an analytic function whose imaginary part is $v = 3 + x^2 - y^2 - \frac{y}{2(x^2+y^2)}$, by Milne Thomson method.

4. Show that $u = \frac{1}{2}\log(x^2+y^2)$ satisfies Laplace's equation. Find its harmonic conjugate.

Practical - 3 : Complex Integrations

1. Evaluate $\int_0^{3+i} z^2\,dz$

   $i)$ along the line $x=3y$ $\qquad ii)$ along the real axis to 3 and then vertically to $3+i$.

   $iii)$ along the parabola $x=3y^2$.

2. Evaluate $\int_C \frac{z+6}{z^2-4}\,dz$, where C is the circle $|z|=1$ by Cauchy's Gourast theorem.

3. Evaluate $\int_C \frac{ze^z}{(z-1)^3}\,dz\ ,\ where\ C\ is\ circle\ |z-1| = 2$, by Cauchy's integral formula.

4. Expand $f(z) = \frac{z^2-1}{(z+2)(z+3)}$ in power of z valid in the regions :

   $i)$ $|z| < 2$, $ii)$ $2 < |z| < 3$, $iii)$ $|z| > 3$.

5. Find Laurent's series for $f(z) = \frac{3z-3}{(2z-3)(z-2)}$ valid for $\frac{1}{2} < |z-1| < 1$.

Practical - 4 : Calculus of Residues

1. If $f(z) = \frac{e^z}{z(z-1)^3}$, Find the sum residues of f(z) at all these poles.

2. Evaluate $\int\limits_{|z|=2} \frac{dz}{z^3(z+4)}$ by Cauchy's residue theorem.

3. Evaluate $\int_C \frac{z^2-2z}{(z+1)^2(z^2+4)}\, dz$ by Cauchy's residue theorem where C is the rectangle formed by the lines $x = \pm 2 , y = \pm 3$

4. Use contour integration to evaluate $\int_0^{2\pi} \frac{1}{3+2\cos\theta}\, d\theta$

5. Use calculus of residue to find value of integral $\int_{-\infty}^{\infty} \frac{x^2-x+2}{x^4+10x^2+9}\, dx$

Practical - 5 : Theory of ordinary differential equations

1 .If f is defined on the rectangle È x È ≤ a , È Y È ≤ b show that the function

f(x,y) = x siny + ycosx satisfies the Lipschitz condition . Find the Lipschitz constant..

2 . Show that Y= 3 e^{2x} + e^{-2x} – 3x is the unique solution of the initial value problem Y' -4Y = 12 X where Y (0) = 4, y'(0) =1.

3 . Show that the functions 1+x, x^2, 1+2x are linearly independent.

4. Show that e^x sin x and e^x Cos x are linearly independent solutions of differential equations Y" -2Y' + 2Y = 0 .

5 . Examine whether e^{2x} and e^{3x} are linearly independent solutions of differential equation Y" -5Y'+6Y =0 or not ?

6. Solve by method of variation of parameters Y" +Y = x .

7. Solve by method of variation of parameters Y" + 9Y = Sec3x .

8. Solve by method of variation of parameters Y" - 3Y'+2Y = $\frac{e^x}{1+e^x}$.

Practical - 6 : Simultaneous Differential Equations

1. Solve $a)\ \frac{dx}{0} = \frac{dy}{-z} = \frac{dz}{y}$ $\quad b)\ \frac{dx}{y^2} = \frac{dy}{x^2} = \frac{dz}{x^2y^2z^2}$ $\quad c)\ \frac{dx}{dt} - y = 1,\ \ \frac{dy}{dt} + x = 1$

2. Solve $a)\ \frac{dx}{y} = \frac{dy}{x} = \frac{dz}{xyz^2(x^2-y^2)}$ $\quad b)\ \frac{dx}{-xy^2} = \frac{dy}{y^3} = \frac{dz}{2xz}$

3. Solve $\frac{dx}{x+y} = \frac{dy}{x-y} = \frac{z.dz}{x^2+2xy-y^2}$

4. Solve $\frac{dx}{y^2(x-y)} = \frac{dy}{z^2(y-z)} = \frac{dz}{x^2(z-x)}$

5. Solve $\frac{dx}{x^2-y^2-z^2} = \frac{dy}{2xy} = \frac{dz}{2xz}$

6. Solve $\frac{dx}{\sin(x+y)} = \frac{dy}{\cos(x+y)} = \frac{dz}{z}$

7. Solve $\frac{dx}{zx-ay} = \frac{dy}{zy+ax} = \frac{dz}{z^2+a^2}$

Practical - 7 : Total (Pfaffian) Differential equations

1. $a)$ Show that the equation $yz^2(x^2-yz)dx + zx^2(y^2-xz)dy + xy^2(z^2-xy)dz = 0$
   Is integrable. Is it exact ? Verify .
   $b)$ Solve: $2xy\,dx - x^2dy + y^2z\,dz = 0$
2. Solve : $\frac{yz\,dx}{x^2+y^2} - \frac{xz\,dy}{x^2+y^2} - \tan^{-1}\left(\frac{y}{x}\right)\,dz = 0$
3. Solve the following by the method of homogeneous differential equations :
   $a)$ $(y^2z - y^3 + x^2y)dx - (x^2z + x^3 - xy^2)dy + (x^2y - xy^2)dz = 0$
   $b)$ $(y^2 + z^2 - x^2)dx - 2xy\,dy - 2xz\,dz = 0$
   $c)$ $(2xz - yz)dx + (2yz - xz)dy - (x^2 - xy + y^2)dz = 0$
4. Solve the following by method of Auxiliary equations :
   $a)$ $(2xz - yz)dx + (2yz - xz)dy - (x^2 - xy + y^2)dz = 0$
   $b)$$(y^2 + z^2 + yz)dx + (z^2 + x^2 + zx)dy + (x^2 + y^2 + xy)dz = 0$
   $c)$ $z(z-y)dx + z(z+x)dy + x(x+y)dz = 0$

Practical - 8 (A) : Beta and Gamma Functions

1.Show that $\int_0^{\infty} x^{m-1}\cos(ax)\,dx = \frac{\Gamma m}{a^m}\cos\left(\frac{m\pi}{2}\right)$

2. a) Show that $\int_0^1 \left[log\left(\frac{1}{x}\right)\right]^{n-1} dx = \Gamma n$

   b) Evaluate $\int_0^1 (x\,logx)^4\,dx$

3. a) Evaluate $\int_0^{\infty} \frac{dx}{3^{4x^2}}$

   b) Show that $\int_0^{\infty} e^{-x^2}dx = \frac{\sqrt{\pi}}{2}$

4. Show that $\int_0^{\pi/2} \sqrt{tan\theta}\,d\theta \quad \int_0^{\pi/2} \sqrt{cot\theta}\,d\theta = \frac{\pi^2}{2}$

5. Evaluate $\int_3^7 (x-3)^{1/4}\,(7-x)^{1/4}\,dx$

6.. Evaluate $\int_{-1}^1 (1+x)^m\,(1-x)^n\,dx$

Practical - 8 (B) : Difference Equations

1. Form the difference equation corresponding to the following general solution :
   $a)$ $y = c_1.3^x + c_2.8^x$ $\quad b)$ $y = (c_1 + c_2n)(-2)^n$ $\quad c)$ $y = c_1x^2 + c_2x + c_3$
2. Show that $y_x = c_1 + c_2.2^x - x$ is a solution of difference equation $y_{x+2} - 3y_{x+1} + 2y_x = 1$
3. Solve the following difference equations :
   $a)$ $y_{x+1} + 3y_x = 0,\ y_0 = 2$ $\quad b)$ $\Delta^3u_n - 5\Delta u_n + 4u_n = 0$
   $c)$ $y_{x+1} = -y_x + 1,\ x = 0,1,2,\ldots\ .and\ y_0 = 1$ $\quad d)$ $y_{x+1} - 3y_x = 1$
4. Solve the following non-homogeneous linear difference equations :
   $a)$ $y_{x+1} - 2y_x = x + 1$ $\quad b)$ $y_{x+2} - 4y_x = 9x^2$
   $c)$ $y_{x+2} - 4y_{x+1} + 4y_x = 3^x + 2^x + 4$ $\quad d)$ $\Delta y_x + \Delta^2 y_x = sinx$
5. Formulate the Fibonnaci difference equation and solve it.

New Equivalences

The equivalences for old courses of S. Y. B. Sc. Mathematics are given as follows:

Sem	Old Course (June 2013)	New equivalent course (June 2016)
I	MTH : 231 – Advanced Calculus	MTH : 231 – Calculus of Several Variables
	MTH : 232(A) – Topics in Algebra	MTH : 232(A) – Algebra
	MTH : 232(B) – Computational Algebra	MTH : 232(B) – Theory of Groups
II	MTH : 241 – Complex Analysis	MTH : 241 – Complex Variables
	MTH : 242(A) – Topics in Differential Equations	MTH : 242(A) – Differential Equations
	MTH : 242(B) – Differential Equations and Numerical Methods	MTH : 242(B) – Differential and Difference Equations
	MTH : 203 – Practical Course based on MTH-231, MTH-232, MTH-241, MTH-242	MTH : 233 – Practical Course based on MTH-241, MTH-242 & MTH : 243 – Practical Course based on MTH-241, MTH-242

NORTH MAHARASHTRA UNIVERSITY, JALGAON

'A' Grade
NAAC Re-Accredited
(3rd Cycle)

STRUCTURE of B.Sc. and SYLLABUS of

F.Y. B. Sc. (MATHEMATICS)

UNDER CHOICE BASED CREDIT SYSTEM (CBCS)

Effective from June 2018

North Maharashtra University, Jalgaon
Scheme for B. Sc. (Mathematics) under Choice Based Credit System pattern with F.Y. B. Sc. Syllabus
Effective from June 2018

Sem.	Course	Paper	Course Code with Title	Credit	No. Periods in Hour /week	No. Periods of 45 min.
I	AECC-1	AECC-1	English/Marathi/Hindi/MIL Communication	2	2	3
	MTHCC- A	Paper - 1	MTH 101: Matrix Algebra	2	2	3
		Paper - 2	MTH 102: Calculus	2	2	3
		Paper - 3	MTH 103 (A): Co-ordinate Geometry **or** MTH 103 (B): Graph Theory	2	2	3
II	AECC-2	AECC-2	English/Marathi/Hindi/MIL Communication	2	2	3
	AECC-3	AECC-3	Environmental Science	2	2	3
	MTHCC- B	Paper-1	MTH 201: Ordinary Differential Equations	2	2	3
		Paper- 2	MTH 202: Theory of Equations	2	2	3
		Paper - 3	MTH 203 (A): Laplace Transform **or** MTH 203 (B): Numerical Analysis	2	2	3
III	MTHCC- C	Paper - 1	MTH 301: Calculus of Several Variables	2	2	3
		Paper - 2	MTH 302: Group Theory	2	2	3
		Paper - 3	MTH 303: Practical paper based on MTH 301 and MTH 302	2	2	3
	SEC-1	SEC-1	Set Theory and Logic	2	2	3
IV	MTHCC-D	Paper - 1	MTH 401: Complex Variables	2	2	3
		Paper - 2	MTH 402: Differential Equations	2	2	3
		Paper - 3	MTH 403: Practical paper based on MTH 401 and MTH 402	2	2	3
	SEC-2	SEC-2	Vector Calculus	2	2	3
V	DSE-1A	Paper - 1	MTH 501: Metric Spaces	2	2	3
		Paper - 2	MTH 502: Real Analysis	2	2	3
		Paper - 3	MTH 503: Practical paper based on MTH 501 and MTH 502	2	2	3
	DSE-2A	Paper - 4	MTH 504: Algebra	2	2	3
		Paper - 5	MTH 505: Integral Transforms		2	3
		Paper - 6	MTH 506: Practical paper based on MTH 504 and MTH 505	2	2	3
	DSE-3A	Paper - 7	MTH 507: Number Theory	2	2	3
		Paper - 8	MTH 508: Programming in C	2	2	3

		Paper - 9	MTH 509: Practical paper based MTH MTH 507 and MTH 508	2	2	3
	SEC-3	SEC-3	Mathematical Modeling	2	2	3
VI	DSE-1B	Paper - 1	MTH 601- Advanced Real Analysis	2	2	3
		Paper - 2	MTH 602 – Measure Theory	2	2	3
		Paper - 3	MTH 603- Practical paper based on MTH 601 and MTH 602	2	2	3
	DSE-2B	Paper - 4	MTH 604- Linear Algebra	2	2	3
		Paper - 5	MTH 605 – Numerical Methods	2	2	3
		Paper - 6	MTH 606- Practical paper based on MTH 604 and MTH 605	2	2	3
	DSE-2C	Paper - 7	MTH 607- Statics	2	2	3
		Paper - 8	MTH 608 – Dynamics	2	2	3
		Paper - 9	Practical paper based on MTH 607 and MTH 608	2	2	3
	SEC-4	SEC-4	Boolean Algebra	2	2	3

MTHCC- A
MTH 101: Matrix Algebra

Course Description:

This course provides an elementary level knowledge of Rank and adjoint of matrix, Applications of matrices to system of linear equations, Eigen values and Eigen vectors of matrices and also the transformation of matrices.

Prerequisite Course(s): 11 and 12 standard Mathematics.

General Objective:

A primary need for the establishment of this course is to understand the basic knowledge and applications of matrices in various fields. So, the main objective is to teach mathematical approaches and models to grow mathematical skill, to improve mathematical thinking and to improve choice making power of the students.

Learning Outcomes:

Upon successful completion of this course the student will be able to:

a) understand concepts on matrix operations and rank of the matrix.
b) understand use of matrix for solving the system of linear equations.
c) understand basic knowledge of the eigen values and eigen vectors.
d) apply Cayley-Hamilton theorem to find the inverse of the matrix.
e) know the matrix transformation and its applications in rotation, reflection, translation.

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UNIT-I: Rank of Matrix: **No. of Periods – 12**

a. Elementary operations on matrices.
b. Adjoint of a matrix & Inverse of a matrix.
c. Existence & uniqueness theorem of inverse of a matrix.
d. Properties of inverse of a matrix, Elementary matrices.
e. Rank and normal form of a matrix, Reduction of a matrix to its normal form, Rank of product of two matrices.

UNIT-II: System of Linear Equations **No. of Periods – 12**

a. A homogeneous and non-homogeneous system of linear equations.
b. Consistency of system of linear equations.
c. Application of matrices to solve the system of linear equations.

UNIT-III: Eigen Values & Eigen Vectors **No. of Periods – 11**

a. Orthogonal Matrices and Properties of Orthogonal Matrices.
b. Characteristic equation, Eigen Values and Eigen Vectors of Matrices.
c. Cayley Hamilton theorem (statement only) and its use to find the inverse of a Matrix.

UNIT-IV: Matrix Transformation **No. of Periods – 10**

a. Two & Three-dimensional Matrix Transform.
b. Application of matrices to Scaling & Shearing.
c. Application of Matrices to Reflection, Rotation & Translation.

REFERENCE BOOKS:

1. Matrix and Linear Algebra, by K. B. Datta, Prentice Hall of India Pvt. Ltd. New Delhi,2000.
2. A Text Book of Matrices, by Shanti Narayan, S. Chand Limited, 2010.
3. Schaum's Outline of Theory and Problems of MATRICES, by Richord Bronson, McGraw-Hill, New York, 1989.
4. Mathematics for Computer Graphics, by Vince, John A., Springer-Verlag London, 2010.
5. Fundamental of Computer Graphics, by Peter Shirley, A. K. Peters, Wellesley, Massachutusetts.
6. Schaum’s Outline of Computer Graphics 2/E 2nd edition, by Zhigang Xiang and Roy A. Plastock, Hall of India Pvt. Ltd., New Delhi, 2015.
7. Matrices & Transformation by Anthony J. Pettofrezzo, Dover Publications, Revised edition, 1978.

MTH 102: Calculus

Course Description: This course provides fundamental knowledge of limits and continuity, Differentiations, Mean value theorem, Rolle's theorem, Cauchy's Mean value theorem and Geometrical interpretations.

Prerequisite Course(s): 11 and 12 standard Mathematics.

General Objective: The basic need of this course is to understand the concepts and applications of calculus. Also, this course will improve problem solving and logical thinking abilities of the students. By learning this course students can use the concepts of calculus to develop different mathematical models.

Learning Outcomes: Upon successful completion of this course the student will be able to:

a) understand basic concepts on limits and continuity.
b) understand use of differentiations in various theorems.
c) know the Mean value theorems and its applications.
d) make the applications of Taylor's, Maclaurin's theorem.
e) know the applications of calculus.

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UNIT-I: Limits and Continuity: **No. of Periods – 12**

a) Epsilon-delta definition of limit of a function.
b) Basic properties of limits, Indeterminate forms & L-Hospitals rule.
c) Continuous functions. Properties of continuous functions on closed and bounded intervals.
d) Theorems on Boundedness of continuous functions, including Intermediate value theorem.
e) Uniform continuity.

UNIT-II: Mean Value Theorems: **No. of Periods – 12**

a) Differentiability.
b) Rolle's Theorem.
c) Lagrange's Mean Value Theorem.
d) Cauchy's Mean Value Theorem.
e) Geometrical interpretation and applications.

UNIT-III: Successive Differentiation: **No. of Periods – 11**

a) The nth derivative of some standard functions: e^{ax+b}, $(ax+b)^m$, x^m, $\frac{1}{(ax+b)}$, $\log(ax+b)$, $\sin(ax+b)$, $\cos(ax+b)$, $e^{ax}\sin(bx+c)$, $e^{ax}\cos(bx+c)$.
b) Leibnitz's theorem & Examples.

UNIT-IV: Applications of Calculus **No. of Periods – 10**

a) Taylor's theorem with Lagrange's form of remainder and related examples.
b) Maclaurin' theorem with Lagrange's form of remainder and related examples
c) Reduction Formulae 1) $\int_0^{\frac{\pi}{2}} sin^n x dx$, 2) $\int_0^{\frac{\pi}{2}} cos^n x dx$, 3) $\int_0^{\frac{\pi}{2}} sin^m x\, cos^n x dx$, 4) $\int \frac{sinnx}{sinx} dx$.

REFERENCE BOOKS:

1. Theory and Problems of Advanced Calculus, by Robert Wrede and Murray R. Spiegel, McGraw-Hill Company, New York, Second Edition, 2002.
2. Text Book on Differential calculus, by Gorakh Prasad, Pothishala Private Ltd., Allahabad, 1959.
3. Integral calculus, by Gorakh Prasad, Pothishala Private Ltd., Allahabad.

MTH 103(A): Coordinate Geometry

Course Description: This course provides an elementary level knowledge of two- and three-dimensional geometries especially sphere, cone and cylinders.

Prerequisite Course(s): 11 and 12 standard Mathematics.

General Objective: General objectives are to study two-dimensional geometry, translation and rotation of axes and its use to convert in standard 2-d forms. Also, to study three-dimensional geometry, Sphere, Cone and Cylinder along with their properties and interpretations.

Learning Outcomes:

Students can visualize geometrical concepts and draw two dimensional figures and can find their standard forms by shifting and rotation of axes. Students also can draw three dimensional figures and their equations particularly Sphere, Cone and Cylinder.

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Unit-I Analytical Geometry: **No. of Periods – 12**

Change of axes, Translation and Rotation, Invariants, Conic section, General equation of second degree in two variables and its reduction to standard form.

Unit-II Sphere: **No. of Periods – 12**

Equation of sphere in different forms, Plane section of sphere, Tangent line and Tangent plane to sphere, Condition of tangency and point of contact, Interpretation of $S + \lambda S' = 0$, and $S + \lambda U = 0$ with usual notations.

Unit-III Cone: **No. of Periods – 12**

Equation of cone with vertex at origin, Equation of cone with vertex at (α, β, γ), Right circular cone, Enveloping cone of sphere, Tangent line and Tangent plane to the cone.

Unit-IV Cylinder: **No. of Periods – 12**

Definition of cylinder, Equation of cylinder, Right circular cylinder, Enveloping cylinder.

REFERENCE BOOKS:

1. The Elements of Coordinate Geometry, By S. L. Loney, Mc-Millan and Company, London, 1895.
2. Text Book of Coordinate Geometry, By Gorakh Prasad and H. C. Gupta, Pothishala Pvt. Ltd. Allahabad, 2000.
3. Analytical Solid Geometry, By Shanti Narayan, S. Chand and Co., 1959.

MTH 103(B): Graph Theory

Unit-I. Graphs **No. of Periods – 12**

Graph, Simple graph, Multigraph, Hand shaking lemma, Types of Graphs, Operations on graphs, Subgraphs, Isomorphism of graphs, Walk, path, cycles (circuits).

Unit-2. Connected Graphs **No. of Periods – 12**

Connected and disconnected Graphs, bridges, Cut vertices, edge connectivity and vertex connectivity, Eulerian graph, Hamiltonian Graph, Planer Graph, Euler's Formula for planer graphs, Kuratowski's two graph, Geometrical dual

Unit-3. Trees and Directed Graphs **No. of Periods – 11**

Definition and some properties of trees, Distance and Centre in a tree, Definitions of Rooted and Binary trees, Spanning trees, Minimal Spanning trees, Directed graphs, some types of digraphs.

Unit-4. Applications of the Graphs **No. of Periods – 10**

Existence of Graphs for given number of Vertices and Edges, Coloring of the graphs, Konigsberg Seven Bridge Problem, Travelling salesman Problem, Dijkstra's algorithm, Warshall's algorithm, formation of flowchart using rooted trees.

Reference books:

1. Graph Theory with Applications to Engineering and Computer science. by Narsingh Deo, Prentice Hall of India Pvt. Ltd. 1979.
   (Unit I: 1.1, 1.4, 1.5, 2.1, 2.2, 2.4, 2.7; Unit II: 2.5, 2.6, 2.9, 5.2, 5.3, 5.4, 5.5, 5.6; Unit III: 3.1, 3.2, 3.4, 3.5, 3.7, 9.1, 9.2; Unit IV: 1.4, 8.1, 1.2, 2.10, 3.4,11.4, 3.5).
2. Theory and Problems of Discreate Mathematics by Seymour Lipschitz and Marc Lars Lipson, Schaum's outline series, McGraw-Hill Ltd., New York, 2007.

MTHCC- B
MTH 201: Ordinary Differential Equations

Course Description: This course provides fundamental knowledge of Ordinary Differential Equations and their applications.

Prerequisite Course(s): 11 and 12 standard Mathematics.

General Objective: The basic need of this course is to understand the different methods of solving differential equations and their applications to solve problems arrives in engineering and technology.

Learning Outcomes: Upon successful completion of this course the student will be able to:

a) understand basic concepts in differential equations.
b) understand method of solving differential equations
c) understand use of differential equations in various fields.

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Unit-I Differential equations of first order and first degree **No. of Periods – 12**

a) Partial derivatives of first order & second orders and Examples.
b) Exact differential equations. Condition for exactness.
c) Integrating factor.
d) Rules for finding integrating factors.
e) Linear differential equations.
f) Bernoulli's Equation. Equation reducible to linear form.

Unit-II Differential equations of first order and higher degree **No. of Periods – 12**

a) Differential equations of first order and higher degree.
b) Equation solvable for p.
c) Equation solvable for y.
d) Equation solvable for x.
e) Clairaut's form.

Unit-III Linear differential equations of second and higher order **No. of Periods – 11**

a) Linear differential equations with constant coefficients.
b) Complementary functions.
c) Particular integrals of f(D)y = X, where X = eax, cos(ax), sin(ax), xn , eaxV, xv with usual notations.

Unit-IV Homogeneous linear differential equations **No. of Periods – 10**

a) Homogeneous linear differential equations (Cauchy's differential equations).
b) Example of Homogeneous linear differential equations.
c) Equations reducible to homogeneous linear differential equations (Legendre's equations)
d) Example of Equations reducible to homogeneous linear differential equations

Reference Books:

1. Introductory Course in Differential Equations, by D. A. Murray, Orient Congman (India) 1967.
2. Differential Equations, by G. F. Simmons, Tata McGraw Hill, 1972.

MTH 202: Theory of Equations

Course Description: This course provides fundamental knowledge of Theory equations.
Prerequisite Course(s): 11 and 12 standard Mathematics.
General Objective: To study

1. Divisibility of numbers and Roots of polynomial equations.
2. Relations between roots and coefficients of polynomials of degree ≤ 4.
3. Roots of cubic equations by using Cardon's method, biquadratic equations by Descarte's method and roots of polynomial equation s by Newton's method.

Learning Outcomes:

Students can find out roots of any equation of degree less than or equal to five. Theory of equations is highly useful in various subjects like algebra, linear algebra, calculus, ordinary and partial differential equations etc.

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Unit-1. Divisibility of Integers **No. of Periods – 12**

Natural numbers. Well ordering principle (statement only). Principle of Mathematical Induction. Divisibility of integers and theorems. Division algorithm. GCD and LCM. Euclidean algorithm. Unique factorization theorem.

Unit-2. Polynomials **No. of Periods – 12**

Revision of Polynomials, Horner's method of synthetic division, Existence and uniqueness of GCD of two polynomials, Polynomial equations, Factor theorem and generalized factor theorem for polynomials, Fundamental theorem of algebra (Statement only), Methods to find common roots of polynomial equation, Descarte's rule of signs, Newton's method of divisors for the integral roots.

Unit-3. Theory of Equations-I **No. of Periods – 11**

Relation between roots and coefficient of general polynomial equation in one variable. Relation between roots and coefficient of quadratic, cubic and biquadratic equations. Symmetric functions of roots.

Unit-4. Theory of Equations –II **No. of Periods – 10**

Transformation of equations. Cardon's method of solving cubic equations. Biquadratic equations. Descarte's method of solving biquadratic equations.

Reference Books:

1. Elementary Number Theory, by David M. Burton, W. C. Brown publishers, Dubuquo lowa 1989.
2. Higher Algebra, by H. S. Hall and S. R. Knight, H. M. Publications 1994.
3. Matrix and Linear Algebra, by K. B. Datta, Prentice Hall of India Pvt. Ltd. New Delhi, 2000.
4. Theory of Equations, by D. R. Sharma, Sharma Publications, Jalandar.

MTH 203(A): Laplace Transform

Course Description: This course provides fundamental knowledge of Laplace transform and their applications in solving differential Equations.
Prerequisite Course(s): 11 and 12 standard Mathematics.
General Objective: The basic need of this course is to understand the concepts and applications of Laplace transforms. The concepts and methods are useful for solving Differential Equations.
Learning Outcomes: Upon successful completion of this course the student will be able to:

a) understand basic concepts on Laplace and Inverse Laplace transforms.
b) Understand convolution theorem.
c) understand use of Laplace transform in solving Differential Equations.

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UNIT-I Laplace Transforms **No. of Periods – 12**

a) Definition and existence of Laplace transform.
b) Laplace transforms of elementary functions and validity. $f(t) = 1, e^{kt}, cosh(kt), sinh(kt), cos(kt), sin(kt), tn\ (t > 0, n > 1)$.
c) Idea of sectionally continuous functions and of exponential order.
d) Properties of Laplace transform.
e) Laplace transforms of derivatives.
f) Laplace transforms of real integrals.
g) Multiplication by tn.
h) Division by t.
i) Definition and simple properties of Beta and Gamma functions (without proof).

UNIT-II Inverse Laplace Transform **No. of Periods – 12**

a) Definition and use of table.
b) Properties of inverse Laplace Transforms :
c) Linearity, first and second translation, Change of scale.
d) Inverse Laplace transforms of derivatives.
e) Inverse Laplace transforms of integrals.
f) Multiplications by s .
g) Division by s.

UNIT-III Convolution Theorem **No. of Periods – 12**

a) Laplace transforms of periodic functions.
b) Convolution theorem (without proof).
c) Evaluation of inverse Laplace transform by convolution theorem.
d) Use of partial fractions.

UNIT-IV Applications to Differential Equations **No. of Periods – 12**

a) Solution of linear differential equation with constant coefficients by using
b) Laplace transforms.
c) Laplace transforms of Heaviside unit step functions.
d) Laplace transforms of Dirac-Delta functions.
e) Applications to Mechanics
f) Applications to electrical circuits,
g) Applications to beams.

Reference books:

1. Theory and problems of Laplace transforms, by Murry R. Spiegel, Schaum's Outline Series, Mc Graw Hill Ltd, New York, 1965.
2. Integral transforms, by A. R. Vasishtha and R. K. Gupta, Krishna Prakashan Media (P) Ltd.

MTH 203(B): Numerical Analysis

Course Description: This course provides fundamental knowledge of different Methods of solution of equations, basics of interpolation and curve fitting for set of data. Also, it provides methods for solving differential equations.

Prerequisite Course(s): 11 and 12 standard Mathematics.

General Objective: The students will be able to understand the basic numerical analysis which is applicable to problems like finding of zeroes of algebraic equations, interpolation, curve fitting and solution of first order differential equations. Students will also understand that when exact solutions are difficult to obtain, then approximate solutions can be obtained by using numerical methods.

Learning Outcomes: Student will be able to:

a) understand basic concepts of methods of solutions of equations viz. bisection, iteration, Newton-Raphson methods and method of false position.

b) understand methods of curve fitting viz. Gauss's forward and backward difference formulae and Lagrange's interpolation formula.
c) use of curve fitting such as least square, polynomial and exponential fittings for set of given data.
d) use Taylor's series, Euler's method. Modified Euler's method., Runge Kutta methods for solving ordinary differential equations.

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UNIT-I Solution of Algebraic and Transcendental Equations **No. of Periods – 12**

Errors and their computation. Absolute, relative and percentage errors. The Bisection method. The iteration method. The method of false position. Newton-Raphson method.

UNIT-II Interpolation **No. of Periods – 12**

Finite differences: Forward differences, backward differences, central differences, Symbolic relations and separation of symbols. Gauss's forward difference formula. Gauss's backward difference formula. Interpolations with unevenly spaced points. Lagrange's interpolation formula. Inverse Lagrange's Formula.

UNIT-III Curve Fitting **No. of Periods – 11**

Least squares curve fitting procedures. Fitting of straight line. Non-linear curve fitting: power function $y = ax + c$. Fitting of polynomial of degree two $y = a + bx + cx^2$. Fitting of exponential function $y = ae^{bx}$.

UNIT-IV Numerical Solutions of Ordinary Differential Equations **No of Periods – 10**

Numerical solution of first order ODE by Taylor's series, Euler's method and Modified Euler's method. Runge–Kutta methods Runge–Kutta second and fourth order formulae.

Reference books:

1. Introductory Methods of Numerical Analysis, by S. S. Sastry, Prentice Hall India Learning Private Limited; Fifth edition, 2012.
2. Introduction to Numerical Analysis, by Carl-Erik Froberg, Addison-Wesley, Second edition, 1979.
3. Numerical Methods by V.N. Vedamurty and N. Ch. S. N. Iyehgar, Vikas Publishing House, India, 1995.
4. Numerical methods for scientific and engineering computation, by M. K. Jain, S. R. K. Iyenger and R. K. Jain. New Age International Publisher Pvt. Ltd., 1999.

KAVAYITRI BAHINABAI CHAUDHARI NORTH MAHARASHTRA UNIVERSITY, JALGAON

STRUCTURE AND SYLLABUS OF

S.Y. B. Sc. (MATHEMATICS)

UNDER CHOICE BASED CREDIT SYSTEM (CBCS)

Effective from June 2019

KAVAYITRI BAHINABAI CHAUDHARI NORTH MAHARASHTRA UNIVERSITY, JALGAON

Syllabus for S. Y. B. Sc. (Mathematics)
Under Choice Based Credit System (CBCS)
Effective from June 2019

The examination pattern is semester system for both the theory and practical papers. Each theory paper is of 100 marks (60 marks for external examination and 40 marks for internal examination) and practical paper is of 100 marks (60 marks for external examination and 40 marks for internal examination). The examination will be conducted at the end of each semester. Period of teaching for each theory paper is 30 clock hours and for practical paper is 60 clock hours.

COURSE STRUCTURE

Sem.	Course	Paper	Course Code with Title	Credits	No. Periods in Hour /week
III	MTHCC- C	Paper - 1	MTH 301: Calculus of Several Variables	2	2
		Paper - 2	MTH 302(A): Group Theory Or MTH -302(B): Theory of Groups and Codes	2	2
		Paper - 3	MTH 303: Practical paper based on MTH 301 and MTH 302	2	4
	SEC-1	SEC-1	MTH 304: Set Theory and Logic	2	2
IV	MTHCC- D	Paper - 1	MTH 401: Complex Variables	2	2
		Paper - 2	MTH 402(A): Differential Equations Or MTH-402 (B): Differential Equations and Numerical Methods	2	2
		Paper - 3	MTH 403: Practical paper based on MTH 401 and MTH 402	2	4
	SEC-2	SEC-2	MTH 404: Vector Calculus	2	2

Syllabus for S.Y. B.Sc. (Mathematics)

SEMESTER – III

MTH -301: Calculus of Several Variables (Period: 30 Clock hours)

Course Description:

This course provides an elementary level knowledge of functions of several variables, their limit continuity, Taylors expansion, differentiation and integration of functions of two or more variables.

Prerequisite Course(s): Preliminary knowledge of real analysis, functions of one variables and calculus.

General Objective:

This is the second course in the calculus series after a course of Calculus in F. Y. B. Sc. for science students. In this course we discuss functions of two and more variables along with their series expansions and extreme values. We also discuss integration techniques as well as applications of integrals.

Learning Outcomes:

Upon successful completion of this course the student will be able to understand:

a) limit and continuity of functions of several variables
b) fundamental concepts of multivariable Calculus.
c) series expansion of functions.
d) extreme points of function and their maximum, minimum values at those points.
e) meaning of definite integral as limit as sums.
f) how to solve double and triple integration and use them to find area by double integration and volume by triple integration.

Unit- 1: Functions of Two and Three Variables **Marks-15**

1.1 Explicit and Implicit Functions
1.2 Continuity
1.3 Partial Derivatives
1.4 Differentiability
1.5 Necessary and Sufficient Conditions for Differentiability
1.6 Partial Derivatives of Higher Order
1.7 Schwarz's Theorem
1.8 Young's Theorem.

Unit-2: Jacobian, Composite Functions and Mean Value Theorems **Marks-15**

2.1 Jacobian (Only for Two and Three Variable)
2.2 Composite Functions (Chain Rule)

2.3 Homogeneous Functions.
2.4 Euler's Theorem on Homogeneous Functions.
2.5 Mean Value Theorem for Function of Two Variables.

Unit -3: Taylor's Theorem and Extreme Values **Marks-15**

3.1 Taylor's Theorem for Function of Two Variables.
3.2 Maclaurin's Theorem for Function of Two Variables.
3.3 Absolute and Relative Maxima & Minima.
3.4 Necessary Condition for Extrema.
3.5 Critical Point, Saddle Point.
3.6 Sufficient Condition for Extrema.

Unit -4: Double and Triple Integrals **Marks-15**

4.1 Double Integrals by Using Cartesian and Polar Coordinates.
4.2 Change of Order of Integration.
4.3 Area by Double Integral.
4.4 Evaluation of Triple Integral as Repeated Integral.
4.5 Volume by Triple Integral.

Recommended Book:

Mathematical Analysis: S.C. Malik and Savita Arora. Wiley Eastern Ltd, New Delhi. 1992 (Chapter 15: Functions of several variables 1, 1.1, 1.2, 1.3, 1.4, 1.6,2, 3, 3.1, 3.2, 4, 4.1, 5, 5.2, 6, 7.2, 9, 9.1, 10, 10.1, 10.2)

Reference Books –

1. Calculus of Several Variables by Schaum's Outline Series.
2. Mathematical Analysis by T. M. Apostol, Narosa Publishing House, New Delhi, 1985

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MTH -302(A): Group Theory (Period: 30 Clock hours)

Course Description:

This course provides an elementary level knowledge of algebraic structure like groups and rings.

Prerequisite Course(s): Preliminary knowledge of sets, functions and binary operations and number systems like Set of integers, rationals, reals and complex.

General Objective:

A primary objective of this course is to understand algebraic structures and their properties. Doing this one can use these structures to solve problems arises in many branches of Mathematics such as theory of equations, theory of numbers, Geometry etc. This enable students to grow their mathematical skill and used them to apply in many branches of science. So, the main objective is to develop and maintain problem-solving skills of the students.

Learning Outcomes:

Upon successful completion of this course the student will be able to:

a) understand group and their types which is one of the building blocks of pure and applied mathematics.
b) understand Lagarnge, Euler and Fermat theorem
c) understand concept of automorphism of groups
d) understand concepts of homomorphism and isomorphism
e) understand basic properties of rings and their types such as integral domain and field.

Unit-1: Groups **Marks-15**

1.1 Definition and Examples of a group.
1.2 Simple Properties of Group.
1.3 Abelian Group.
1.4 Finite and Infinite Groups.
1.5 Order of a Group.
1.6 Order of an Element and Its Properties.

Unit-2: Subgroups **Marks-15**

2.1 Definition and Examples of Subgroups.
2.2 Simple Properties of Subgroup.
2.3 Criteria for a Subset to be a Subgroup.
2.4 Cyclic Groups
2.5 Normal subgroups and Coset Decomposition.
2.6 Lagrange's Theorem for Finite Group.
2.7 Euler's Theorem and Fermat's Theorem.

Unit-3: Homomorphism and Isomorphism of Groups **Marks-15**

4.1 Definition and Examples of Group Homomorphism.
4.2 Properties of Group Homomorphism.
4.3 Kernel of a Group Homomorphism and it's Properties.
4.4 Definition and Examples of Isomorphism.
4.5 Definition and Examples of Automorphism of Groups.
4.6 Properties of Isomorphism of Groups.

Unit -4: Rings **Marks-15**

4.1 Definition and Simple Properties of a Ring.
4.2 Commutative Ring, Ring with unity, Boolean Ring.
4.3 Ring with zero divisors and without zero Divisors.
4.4 Integral Domain, Division Ring and Field. Simple Properties.

Recommended Book: -

1. University Algebra: N. S. Gopalakrishnan, New age international publishers, 2018. (Chapter 1: 1.3, 1.4, 1.5, 1.6,1.7, 1.8, 1.9)

Reference Books: -

1. Topics in Algebra: I. N. Herstein (John Wiley and Sons).
2. A first Course in Abstract Algebra: J. B. Fraleigh (Pearson).
3. A course in Abstract Algebra: Vijay K. Khanna and S. K. Bhambri, Vikas Publishing House Pvt. Ltd., Noida.

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MTH -302(B): Theory of Groups and Codes (Period: 30 Clock hours)

Course Description:

This course provides an elementary level knowledge of algebraic structure like groups and codes.

Prerequisite Course(s): Preliminary knowledge of Sets, functions and binary operations and number systems like Set of integers, rationals, reals and complex.

General Objective:

A primary need for the establishment of this course is to understand algebraic structures and their properties. Upon studying this one can use these sutures to solve problems arises in many branches of Mathematics and computer science such as theory of equations, theory of numbers, Geometry, theory of computations, cryptography etc. This enable students to grow their mathematical skill and used them to apply in many other branches of science and technology.

Learning Outcomes:

Upon successful completion of this course the student will be able to:

a) understand group structures which is useful to understanding ideas of modern mathematics.
b) understand solutions to polynomial equations
c) understand permutation groups
d) understand concepts of homomorphisms and isomorphisms
e) Students will understand basic concepts in codding theory.

Unit-1: Groups **Marks-15**

1.1 Definition and Examples of a group.
1.2 Simple Properties of Group.
1.3 Abelian Group.
1.4 Finite and Infinite Groups.
1.5 Order of a Group.
1.6 Order of an Element and Its Properties.

Unit-2: Subgroups **Marks-15**

2.1 Definition and Examples of Subgroups.
2.2 Simple Properties of Subgroup.
2.3 Criteria for a Subset to be a Subgroup.

2.4 Cyclic Groups
2.5 Normal subgroups and Coset Decomposition.
2.6 Lagrange's Theorem for Finite Group.
2.7 Euler's Theorem and Fermat's Theorem.

Unit-3: Homomorphism and Isomorphism of Groups Marks-15

3.1 Definition and Examples of Group Homomorphism.
3.2 Properties of Group Homomorphism.
3.3 Kernel of a Group Homomorphism and it's Properties.
3.4 Definition and Examples of Isomorphism.
3.5 Definition and Examples of Automorphism of Groups.
3.6 Properties of Isomorphism of Groups.

Unit -4: Group Codes Marks 15

4.1 Message, Word, (m, n)- Encoding Function, Code Words.
4.2 Detection of k or fewer errors, Weight, Parity Check Code
4.3 Hamming Distance, Properties of the Distance Function, Minimum Distance of an encoding function.
4.4 Group Codes.
4.5 (n, m)- Decoding function, Maximum Likelihood Decoding Function.
4.6 Decoding procedure for a Group Code given by a Parity Check Matrix.

Recommended Book: -

1. University Algebra: N. S. Gopalakrishnan, New age international publishers, 2018. (Chapter 1: 1.3, 1.4, 1.5, 1.6,1.7, 1.8, 1.9)
2. Discrete Mathematical Structures: Bernard Kolman, Robert C. Busby and Ross (Prentice Hall of India New Delhi, Eastern Economy Edition).

Reference Books: -

1. Topics in Algebra: I. N. Herstein (John Wiley and Sons).
2. A first Course in Abstract Algebra: J. B. Fraleigh (Pearson).

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MTH-303: Practical Course Based on MTH-301 and MTH-302

(Period: 60 Clock hours)

Practical No	Title of the Practical
1	Functions of two and three Variables
2	Jacobian, Composite Functions and Mean Value Theorems
3	Taylor's Theorem and Extreme Values
4	Double and Triple Integrals
5	Groups
6	Subgroups
7	Homomorphism and Isomorphism of Groups
8(A)	Rings
8(B)	Group Codes

List of Practical Problems

Practical 1: Functions of Two and Three Variables

1. Evaluate the limit, if it exists, for the following function

$$f(x,y) = \begin{cases} \dfrac{x^2 y}{x^4 + y^2} & , if\ x^4 + y^2 \neq 0 \\ 0 & , if\ x = y = 0. \end{cases}$$

2. Let $f(x,y) = x\sin\frac{1}{x} + y\sin\frac{1}{y}$, $xy \neq 0$. Show that $\lim_{(x,y)\to(0,0)} f(x,y) = 0$.

3. Let $f(x,y) = \frac{x^2y^2}{x^4+y^4-x^2y^2}$, $(x,y) \neq (0,0)$. Verify that both the repeated limits exist and are equal, but simultaneous limit does not exist.

4. Show that the function

$$f(x,y) = \begin{cases} \dfrac{xy}{\sqrt{x^2 + y^2}} & , if\ (x,y) \neq (0,0) \\ 0 & , if\ (x,y) = (0,0) \end{cases}$$

is continuous at the origin.

5. Let $f(x,y) = \begin{cases} \frac{xy}{x^2+y^2} & , if\ (x,y) \neq (0,0) \\ 0 & , if\ (x,y) = (0,0) \end{cases}$

Show that both the first order partial derivatives exist at $(0, 0)$, but the function is not continuous thereat.

6. Discuss the continuity and differentiability at the origin of the function

$$f(x,y) = \begin{cases} \frac{xy}{x^2+y^2} & , if\ (x,y) \neq (0,0) \\ 0 & , if\ (x,y) = (0,0) \end{cases}$$

7. Let $f(x,y) = \begin{cases} x^2\tan^{-1}\left(\frac{y}{x}\right) - y^2\tan^{-1}\left(\frac{x}{y}\right) & , if\ (x,y) \neq (0,0) \\ 0 & , if\ (x,y) \neq (0,0) \end{cases}$

Show that $f_{xy}(0,0) \neq f_{yx}(0,0)$.

8. Show that for the function

$$f(x,y) = \begin{cases} \dfrac{xy}{x^2 + y^2} & , if\ (x,y) \neq (0,0) \\ 0, & if\ (x,y) = (0,0) \end{cases}$$

$f_{xy}(0,0) = f_{yx}(0,0)$, even though the conditions of Schwarz's theorem and Young's theorem are not satisfied.

9. Using differentials find approximate value of $\sqrt{(1.02)^2 + (1.97)^3}$.
10. Using differentials find approximate value of $(3.9)^2(2.05) + (2.05)^3$.

Practical 2: Jacobian, Composite functions and Mean value theorem

1. If $u = \cos x$, $v = \sin x \cos y$, $w = \sin x \sin y \cos z$, then show that
$$\frac{\partial(u,v,w)}{\partial(x,y,z)} = (-1)^3 sin^3 x\ sin^2 y \sin z$$
2. If $z = f(x,y) = \tan^{-1}\left(\frac{x}{y}\right)$, $x = u + v$, $y = u - v$, then show that $\frac{\partial z}{\partial u} + \frac{\partial z}{\partial v} = \frac{u-v}{u^2+v^2}$.
3. If $u = f(e^{y-z}, e^{z-x}, e^{x-y})$, then prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$
4. If z is function of x and y and if $x = e^u + e^{-v}$, $y = e^{-u} - e^v$, then prove that $\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y}$.
5. If $z = f(u,v)$, where $u = 2x - 3y$ and $v = x + 2y$, then prove that $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 3\frac{\partial z}{\partial v} - \frac{\partial z}{\partial u}$.
6. If $u = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$, then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$. Hence, deduce that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = (1 - 4sin^2 u)sin2u$.
7. If $u = \sin^{-1}\left(\frac{x^2+2xy}{\sqrt{x-y}}\right)^{\frac{1}{5}}$, then find the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ and $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2}$.
8. If $u = \tan^{-1}\left(\frac{\sqrt{x^2+y^2}}{x-y}\right)$, then find the value of $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2}$.
9. Let$f(x,y) = x^2 y + 2xy^2$. Find the quadratic equation in θ by applying the mean value theorem applied to the line segment joining the points (1,2) to (3,3).
10. Let$f(x,y) = x^3 - xy^2$. Show that θ used in the mean value theorem applied to the points (2,1) and (4,1) satisfies the quadratic equation $3\theta^2 + 6\theta - 4 = 0$.

Practical 3: Taylor's theorem and Extreme values

1. Use Taylor's theorem of suitable order to expand $\sin x \sin y$ in the form $xy - \frac{1}{6}\{(x^3 + 3xy^2)\cos\theta x \sin\theta y + (y^3 + 3x^2 y)\sin\theta x \cos\theta y\}$,$0 < \theta < 1$.
2. Show that the expansion of $\sin(xy)$ in powers of $(x-1)$ and $\left(y - \frac{\pi}{2}\right)$ upto and including second degree terms is
$$1 - \frac{1}{8}\pi^2(x-1)^2 - \frac{1}{2}\pi(x-1)\left(y - \frac{\pi}{2}\right) - \frac{1}{2}\left(y - \frac{\pi}{2}\right)^2.$$

3. Using Maclaurin's expansion, prove that $e^{ax}\cos by = 1 + ax + \frac{a^2x^2 - b^2y^2}{2!} + \frac{a^3x^3 - 3ab^2xy^2}{3!}$.

4. Expand $e^x tan^{-1}y$ about (1 ,1) up to the second degree in powers of $(x-1)$and $(y-1)$.

5. Find maxima and minima of the function $f(x,y) = x^3 + y^3 - 3x - 12y + 20$.

6. Discuss the extreme values of the function $f(x,y) = 2(x^2 - y^2) - x^4 + y^4$.

7. Investigate maximum and minimum values of $f(x,y) = (x + y - 1)(x^2 + y^2)$.

8. Find the extreme values of $f(x,y) = xy(a - x - y)$.

9. Find the least value of the function $f(x,y) = xy + \frac{50}{x} + \frac{50}{y}$.

Practical –4: Double and Triple Integrals

1. Using double integration, find the area between the parabola $y^2 = 4ax$ and $x^2 = 4ay$.
2. Find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, using double integral.
3. Evaluate $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z}\, dxdydz$.
4. Using triple integration, find the volume of the sphere of radius a.
5. Evaluate $\int_0^1 \int_0^2 \int_0^3 (x + y + z)\, dxdydz$.
6. Change the order of integration in $\int_0^4 \int_0^{\sqrt{4x-x^2}} f(x,y)dxdy$.
7. Draw a sketch of the region of integration
   i) $\int_0^4 \int_0^{\sqrt{25-x^2}} f(x,y)dxdy$.
   ii) $\int_{-1}^3 \int_{x^2}^{x+3} f(x,y)dxdy$.
8. Evaluate $\int\int ydxdy$ over the area bounded by $y = x^2$ and $x + y = 2$.

Practical - 5: Groups

1. Let $\mathbb{Q}^+$ denotes the set of all positive rational numbers and for any $a\,,b \in \mathbb{Q}^+$, define $a * b = \frac{ab}{3}$. Show that $(\mathbb{Q}^+\,, *)$ is an abelian group.
2. Let $G = \{(a\,,b) : a\,,b \in \mathbb{R},\ a \neq 0\}$. Show that $(G\,,\odot)$ is a non-abelian group, where $(a\,,b)\odot(c\,,d) = (ac\,,ad + b)$.
3. Let G be a group and $a \in G\ ,n \in \mathbb{N}$. Show that $a^n = e$ if and only if $o(a)|\,n$.
4. Show that a group G is abelian if and only if $(ab)^2 = a^2b^2\ \forall\, a\,,b \in G$.

5. In the group ($\mathbb{Z}_7$, $\times_7$), find (i) $(\bar{3})^2$ ii) $(\bar{4})^{-3}$ iii) $o(\bar{3})$ iv) $o(\bar{4})$
6. In the group ($\mathbb{Z}_{11}$, $\times_{11}$), find (i) $(\bar{4})^3$ ii) $\left(\bar{5}\right)^2$ iii) $o(\bar{9})$ iv) $o(\bar{7})$
7. Show that $G = \mathbb{R} - \{1\}$ is an abelian group under the binary operation $a * b = a + b - ab$, $\forall\, a, b \in G$
8. Prove that $G = \{\begin{bmatrix} x & x \\ x & x \end{bmatrix}$: x is a non-zero real number$\}$ is a group under matrix multiplication.
9. If G is a group such that $a^2 = e, \forall\, a, b \in G$, then show that G is abelian.
10. If in a group G, $a^5 = e$ and $aba^{-1} = b^2, \forall\, a, b\ G$, then find order of an element b.

Practical - 6: Subgroups

1. If G is a group, then show that the center of $G, Z(G)$, is a subgroup of G, where $Z(G) = \{x \in G : xa = ax, \ \forall a \in G\}$.
2. Show that ($\mathbb{Z}_7$, $\times_7$)is a cyclic group. Find all its generators, all its proper subgroups and order of every element.
3. Let $G = \{1, -1, i, -i, j, -j, k, -k\}$ be a group under multiplication and $H = \{1, -1, i, -i\}$ be its subgroup. Find all the left and right cosets of H in G.
4. Let A and B be two subgroups of a finite group G whose orders are relatively prime. Show that $A \cap B = \{e\}$.
5. Show that every proper subgroup of a group of order 77 is cyclic.
6. Find the remainder obtained when 15^{27} is divided by 8.
7. Find the remainder obtained when 33^{19} is divided by 7.
8. Let G be a group of all non-zero complex numbers under multiplication. Show that $H = \{a + ib : a^2 + b^2 = 1\}$ is a subgroup of G.
9. If H is subgroup of a group G and if the normalizer of H, $N(H) = \{g \in G : gHg^{-1} = H\}$, then prove that (a) $N(H)$ is a subgroup of G and (b) H is a normal subgroup of $N(H)$.
10. If G is a group and H is a subgroup of index 2 in G, then prove that H is a normal subgroup of G.

Practical – 7: Homomorphism and Isomorphism of Groups

1. Let $G = \{A : A$ is $n \times n$ matrix over $\mathbb{R}$ and $|A| \neq 0\}$, the group of non-singular matrices of order n over $\mathbb{R}$ under matrix multiplication and let $\mathbb{R}^* = \mathbb{R} - \{0\}$, be the group of nonzero real numbers under multiplication. Define $f : G \to \mathbb{R}^*$ by $f(A) = |A|$, for all $A \in G$. Show that f is an onto group homomorphism and find its kernel.
2. If $G_1 = \{1, -1, i, -i\}$ is a group under multiplication and $G_2 = \{2, 4, 6, 8\}$ is a group under multiplication modulo 10, then show that G_1 and G_2 are isomorphic.
3. Let G be a group and $a \in G$. Show that $f_a : G \to G$ defined by $f_a(x) = axa^{-1}$, for all $x \in G$ is an automorphism.

4. Let G be a group and $f : G \to G$ be a map defined by $f(x) = x^{-1}$, for all $x \in G$. Prove that

(a) If G is abelian, then f is an isomorphism.

(b) If f is a group homomorphism, then G is abelian.

5. Let $G = \{a, a^2, a^3, \dots, a^{11}, a^{12} = e\}$ be a cyclic group of order 12 generated by a. Show that $f : G \to G$ defined by $f(x) = x^4$, $\forall\, x \in G$ is a group homomorphism. Find the kernel of f.

6. Let f and g be group homomorphisms from the group G into G. Show that $H = \{x \in G : f(x) = g(x)\}$ is a subgroup of G.

7. Prove that the mapping $f: C \to C_0$ such that $f(z) = e^z$ is a homomorphism of the additive group of complex numbers onto the multiplicative group of non-zero complex numbers. What is the kernel of f?

8. Let G be a group of all matrices of the type $\left\{\begin{bmatrix} a & b \\ -b & a \end{bmatrix} : a, b \in G \text{ and } a^2 + b^2 = 1\right\}$ under matrix multiplication and G' be a group of non-zero complex numbers under multiplication. Show that $f : G \to G'$ defined by $f\left(\begin{bmatrix} a & b \\ -b & a \end{bmatrix}\right) = a + ib$, is an isomorphism.

Practical – 8(A): Rings

1. (a) Show that $\mathbb{Z}_7 = \{\overline{0}, \overline{1}, \overline{2}, \overline{3}, \overline{4}, \overline{5}, \overline{6}\}$ forms a ring under addition and multiplication modulo 7.

(b) In the ring $(\mathbb{Z}_{10}, +_{10}, \times_{10})$, find all divisors of zero.

2. Show that $\mathbb{Z}[i] = \{a + ib : a, b \in \mathbb{Z}\}$, the set of Gaussian integers, forms an integral domain under usual addition and multiplication of complex numbers.

3. Show that $R = \{a + b\sqrt{2} : a, b \in \mathbb{Z}\}$ is an integral domain under usual addition and multiplication.

4. In the ring $(\mathbb{Z}_7, +_7, \times_7)$, find (i) $-(\overline{4} \times_7 \overline{6})$; (ii) $\overline{3} \times_7 (\overline{-6})$; (iii) $(\overline{-5}) \times_7 (\overline{-5})$ (iv) Units in $\mathbb{Z}_7$; (v) additive inverse of $\overline{6}$; (vi) zero divisors. Is $\mathbb{Z}_7$ a field or an integral domain? Justify.

5. Let $\mathbb{R}$ be the set of all real numbers. Show that $\mathbb{R} \times \mathbb{R}$ forms a field under addition and multiplication defined by $(a, b) + (c, d) = (a + c, b + d)$ & $(a, b) . (c, d) = (ac - bd, ad + bc)$.

6. If p is a prime number, then show that $\mathbb{Z}_p$ is an integral domain.

7. Which of the following rings are integral domains? (i) $\mathbb{Z}_{187}$; (ii) $\mathbb{Z}_{61}$; (iii) $\mathbb{Z}_{2\times 2}$. (iv) $(\mathbb{Z}, +, \cdot)$.

Practical – 8(B): Group Codes

1. Consider the (3,8) encoding function $e : B^3 \to B^8$ defined by $e(000) = 00000000$, $e(001) = 10111000$, $e(010) = 00101101$, $e(011) = 10010101$, $e(100) = 10100100$, $e(101) = 10001001$, $e(110) = 00011100$, $e(111) = 00110001$.
   (a) Find the minimum distance of e.
   (b) How many errors will e detect?

2. Show that the (3,6) encoding function $e : B^3 \to B^6$ defined by $e(000) = 000000$, $e(001) = 001100$, $e(010) = 010011$, $e(011) = 011111$, $e(100) = 100101$, $e(101) = 101001$, $e(110) = 110110$, $e(111) = 111010$is a group code. Also find the minimum distance of e.

3. Compute: (a) $\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \oplus \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} * \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

4. Let H $= \begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ be a parity check matrix. Determine the $(2,5)$ group code $eH : B^2 \to B^5$.

5. Consider the parity check matrix: H $= \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Decode the following words relative to a maximum likelihood decoding function associated with eH : $a)$ 10100 $b)$ 01101 $c)$ 11011

6. Consider the parity check matrix: H $= \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Determine the coset leaders for $N = eH(B^3)$. Also compute the Syndrome for each coset leader and decode the code 001110 relative to maximum likelihood decoding function.

7. Let the $(9,3)$ decoding function $d : B^9 \to B^3$ be defined by $d(y) = Z1Z2Z3$, where $Z_i = 1$, if $\{yi, yi + 3, yi + 6\}$ has at least two 1's
   $= 0$, if $\{yi, yi + 3, yi + 6\}$ has less than two 1's, i = 1, 2, 3.

8. If $y \in B^9$, then determine $d(y)$, where (i) $y = 101111101$ (ii) $y = 100111100$.

Note: Practical problems based on each unit are not limited to the given ones, but any other related challenging and application-oriented problems may also be evaluated in the practical sessions.

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SEC -1

MTH 304: Set Theory and logic (Period: 30 Clock hours)

Course Description:

This course is an elementary skill development course for S. Y. B.Sc. students.

Prerequisite Course(s): Secondary school level knowledge of elementary mathematics.

General Objective: The general objectives are to acquire concepts of sets, relations, countable and uncountable sets; statements and truth values; concept of tautology, contradiction and quantifiers.

Learning Outcomes:

a) Uses of the language of set theory, designining issues in different subjects of mathematics
b) understand the issues associated with different types of finite and infinite sets via countable uncountable sets
c) knowledge of the concepts and methods of mathematical logic, set theory, relation calculus, and concepts concerning functions which are included in the fundamentals of various disciplines of mathematics
d) understanding the role of propositional and predicate calculus
e) able to provide the logical mathematical reasoning, formulate theorems and definitions

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Unit-1: Sets and Subsets **Marks-15**

1.1 Finite Set and Infinite set

1.2 Equality of two Sets,

1.3 Null Set, Subset, Proper subset, Symmetric difference of two sets

1.4 Universal set, Power set, Disjoint sets,

1.5 Operation on sets: Union and Intersection

1.6 Venn diagram

1.7 Equivalent sets

1.8 Countable and uncountable sets

Unit-2: Relations and Functions **Marks-15**

1.1 Product of sets

1.2 Relations, Types of relations, Reflexive, Symmetric, Transitive relations and Equivalence relations
1.3 Function, Types of functions, One-one, Onto, Even, Odd and Inverse function
1.4 Composite functions

Unit-3: Algebra of Propositions **Marks-15**

2.1 Statements, Conjunction, Disjunction.
2.2 Negation, Conditional and Bi-Conditional statements, Propositions.
2.3 Truth table, Tautology and Contradiction.
2.4 Logical equivalence, Logical equivalent statements.

Unit-4: Quantifiers **Marks-15**

3.1 Propositional functions and Truth sets.
3.2 Universal quantifier, Existential quantifier.
3.3 Negation of proposition which contain quantifiers, Counter examples.

Recommended book:

1. Set Theory and Related Topics by Schaum's outline Series (Chapter1, chapter 4, chapter 6: 6.2, 6.3, chapter 10)

Reference Books:

1. R.R.Halmons, Naïve Set Theory, Springer, 1974
2. E. Kamke, Theory of Sets, Dover Publishers,1950

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SEMESTER – IV

MTH -401: Complex Variables (Period: 30 Clock hours)

Course Description:

This course will improve basic and intermediate level knowledge of a special type of number system namely complex numbers and also discusses complex valued function with their integrations.

Prerequisite Course(s): Basic knowledge of Sets, functions, real valued functions, their limits and continuity and integrations.

General Objective:

A primary objective of this course is to make students aware of generalization of real number system and calculus. Analyticity and complex integrations are useful for applications. This course improves mathematical skill and ability to solve various integrations.

Learning Outcomes:

a) The course is aimed to introduce the theory for functions of complex variables
b) Students will understand the concept of analytic function
c) Students will understand the Cauchy Riemann Equations
d) Students will understand harmonic functions
e) Students will understand complex integrations
f) Students will understand calculus of residues.
g) Students will acquire the skill of contour integrations.

Unit-1: Complex numbers **Marks-15**

1.1 Complex numbers, modulus and amplitude, polar form
1.2 Triangle inequality and Argand's diagram
1.3 DeMoivre's theorem for rational indices and applications
1.4 n^{th} roots of a complex number
1.5 Elementary functions: Trigonometric functions, Hyperbolic functions of a complex variables (definitions only).

Unit-2: Functions of complex variables **Marks-15**

2.1 Limits, Continuity and Derivative.
2.2 Analytic functions, A Necessary and sufficient conditions for analytic functions.
2.3 Cauchy Riemann equations.
2.4 Laplace equations and Harmonic functions
2.5 Construction of analytic functions

Unit-3: Complex integrations **Marks-15**

3.1 Line integral and theorems on it.

3.2 Statement and verification of Cauchy-Gaursat's Theorem.
3.3 Cauchy's integral formulae for $f(a), f'(a)$ and $f^n(a)$
3.4 Taylor's and Laurent's series.

Unit-4: Calculus of Residues **Marks-15**

4.1 Zeros and poles of a function.
4.2 Residue of a function
4.3 Cauchy's residue theorem
4.4 Evaluation of integrals by using Cauchy's residue theorem
4.5 Contour integrations of the type $\int_0^{2\pi} f(\cos\theta, \sin\theta)d\theta$ and $\int_{-\infty}^{\infty} f(x)dx$

Recommended book:

1. Complex Variables and Applications; J. W. Brownand, R. V. Churchill. 7th Edition. (McGraw-Hill) (Capter1, chapter 2, chapter 3, chapter 4, chapter 6)

Reference Books:

1. Theory of Functions of Complex Variables: Shanti Narayan, S. Chand and Company New Delhi.
2. Complex variables: Schaum's Outline Series.

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MTH-402(A): Differential Equations (Period: 30 Clock hours)

Course Description:

This course is of primary nature and here we introduce the students how the Differential Equations are formed and how to solve them using various methods.

Prerequisite Course(s): Basic knowledge of Real and complex valued functions, differentiations and integrations.

General Objective:

The main objective of this program is to cultivate mathematical aptitude among the students and nurture their interest towards problem solving aptitude by introducing various methods of solution of differential equations.

Learning Outcomes:

a) Students will aware of formation of differential equations and their solutions
b) Students will understand the concept of Lipschitz condition
c) Students will understand method of variation of parameters for second order L.D.E.
d) Students will understand simultaneous linear differential equations and method of their solutions
e) Students will understand Pfaffian differential equations and method of their solutions
f) Students will understand difference equations and their solutions

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Unit-1: Theory of ordinary differential equations **Marks-15**

1.1 Lipschitz condition
1.2 Existence and uniqueness theorem
1.3 Linearly dependent and independent solutions
1.4 Wronskian definition
1.5 Linear combination of solutions
1.6 Theorems on i) Linear combination of solutions ii) Linearly independent solutions iii) Wronskian is zero iv) Wronskian is non-zero
1.7 Method of variation of parameters for second order L.D.E.

Unit-2: Simultaneous Differential Equations **Marks-15**

2.1 Simultaneous linear differential equations of first order
2.2 Simultaneous D.E. of the form $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$.
2.3 Rule I: Method of combinations
2.4 Rule II: Method of multipliers
2.5 Rule III: Properties of ratios
2.6 Rule IV: Miscellaneous

Unit-3: Total Differential or Pfaffian Differential Equations **Marks-15**

3.1 Pfaffian differential equations
3.2 Necessary and sufficient conditions for the integrability
3.3 Conditions for exactness
3.4 Method of solution by inspection
3.5 Solution of homogenous equation

Unit-4: Difference Equations **Marks-15**

4.1 Introduction, Order of difference equation, degree of difference equations
4.2 Solution to difference equation and formation of difference equations
4.3 Linear difference equations, Linear homogeneous difference equations with constant coefficients
4.4 Non-homogenous linear difference equation with constant coefficients

Recommended books:

1. Ordinary and Partial Differential Equation by M. D. Rai Singhania, S. Chand & Co. 18th Edition. (Chapter 1 and Chapter 2)
2. Numerical Methods by V. N. Vedamurthy and N. Ch. S. N. Iyengar, Vikas Publishing House, New Delhi. (Chapter 10).

Reference Book:

1. Introductory course in Differential Equations by D. A. Murray, Longmans Green and co. London and Mumbai, 5th Edition 1997.

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MTH-402 (B): Differential Equations and Numerical Methods (Period: 30 Clock hours)

Unit-1 : Theory of ordinary differential equations **Marks-15**

1.1 Lipschitz condition
1.2 Existence and uniqueness theorem
1.3 Linearly dependent and independent solutions
1.4 Wronskian definition
1.5 Linear combination of solutions
1.6 Theorems on i) Linear combination of solutions ii) Linearly independent solutions
iii) Wronskian is zero iv) Wronskian is non-zero
1.7 Method of variation of parameters for second order L.D.E.

Unit-2 : Simultaneous Differential Equations **Marks-15**

2.1 Simultaneous linear differential equations of first order
2.2 Simultaneous D.E. of the form $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$.
2.3 Rule I: Method of combinations
2.4 Rule II: Method of multipliers
2.5 Rule III: Properties of ratios
2.6 Rule IV: Miscellaneous

Unit-3 : Total Differential or Pfaffian Differential Equations **Marks-15**

3.1 Pfaffian differential equations
3.2 Necessary and sufficient condition for integrability
3.3 Conditions for exactness
3.4 Method of solution by inspection
3.5 Solution of homogenous equation

Unit-4 : Numerical differentiation **Marks-15**

4.1 Numerical Differentiation
4.2 Derivatives using Newtons forward interpolation formula
4.3 Derivatives using Newtons backward interpolation formula
4.4 Derivatives using Stirling's interpolation formula
4.5 Maxima and minima

Recommended books: -

1. Ordinary and Partial Differential Equation by M. D. Rai Singhania, S. Chand & Co. 18th Edition. (Chapter 1 and Chapter 2)
2. Numerical Methods by Dr. V. N. Vedamurthy and Dr. N. Ch. S. N. Iyengar, Vikas Publishing (Chapter 9)

Reference Books:

1. Introductory methods of Numerical Analysis, S.S. Sastry, Prentice Hall India, 12 th edition, New Delhi.
2. Differential equations, G.F. Simmons, Tata Mcgrawhill, 1972.

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MT-403: Practical course based on MTH-401, MTH-402
(Period: 60 Clock hours)

Practical No	Title of the Practical
1	Complex Numbers
2	Function of Complex Variable
3	Complex Integration
4	Calculus of Residues
5	Theory of ordinary differential equations
6	Simultaneous Differential Equations
7	Total (Pfaffian) Differential Equations
8(A)	Difference Equations
8(B)	Numerical Differentiation

List of Practicals

Practical-1: Complex Numbers

1. Find the Modulus and principle value of the argument of $\frac{(1+i\sqrt{3})^{13}}{(\sqrt{3}-i)^{11}}$.
2. If z_1, z_2, z_3 represents the vertices of an equilateral triangle, then prove that $z_1^2 + z_2^2 + z_3^2 = z_1z_2 + z_2z_3 + z_3z_1$.
3. If $cos\alpha + cos\beta + cos\gamma = 0$ and $sin\alpha + sin\beta + sin\gamma = 0$, then show that
   i. $cos3\alpha + cos3\beta + cos3\gamma = 3\cos(\alpha+\beta+\gamma)$ and
   $sin3\alpha + sin3\beta + sin3\gamma = 3\sin(\alpha+\beta+\gamma)$
   ii. $cos2\alpha + cos2\beta + cos2\gamma = 0$ and
   $sin2\alpha + sin2\beta + sin2\gamma = 0$
4. Find all the values of $(1+i)^{\frac{1}{5}}$. Show that their continued product is $1+i$.
5. Solve the equation $x^8 - x^4 + 1 = 0$.
6. Determine the region in the Z-plane represented by $|z-3| + |z+3| = 10$.
7. Using De Moivre's theorem express $\cos^6\theta$ in terms of cosines of multiple angles.
8. If $|z_1| = |z_2| = |z_3| = 5$ and $\overline{z_1} + \overline{z_2} + \overline{z_3} = 0$, then prove that $\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3} = 0$.

Practical-2: Functions of Complex Variable

1. Evaluate: $\lim_{z \to (1+i)} \frac{z^4+4}{z-1-i}$.
2. If $f(z) = \frac{3z^4-2z^3+8z^2-2z+5}{z-i}$, $z \neq i$ is continuous at $z = i$, then find the value of $f(i)$.
3. Find an analytic function $f(z) = u + iv$ and express it in terms of z, if $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$

4. Find an analytic function $f(z) = u + iv$ whose imaginary part is $v = e^x(xsiny + ycosy)$ using Milne Thomson Method.
5. Show that the real and imaginary parts of the function e^z satisfy C-R equations and they are harmonic.
6. Show that $u = \frac{1}{2}\log(x^2 + y^2)$ satisfies Laplace Equation. Find its harmonic conjugate.
7. If $f(z)$ is an analytic function with constant modulus, then show that $f(z)$ is a constant function.
8. Evaluate $\lim_{z \to e^{\frac{i\pi}{3}}} \frac{(z-e^{\frac{i\pi}{3}})z}{Z^3+1}$.

Practical-3: Complex Integration

1. Evaluate $\int_C^{\cdot} (y - x - 3x^2 i)dz$, where C is :
   i. The straight-line joining $z = 0$ to $z = 1 + i$
   ii. The straight-line joining $z = 0$ to $z = i$ first and then from $z = i$ to $z = 1 + i$.
2. Use the Cauchy Goursat theorem to obtain the value of $\int_C^{\cdot} e^z \; dz$, where C is the circle $|z| = 1$ and hence deduce the following:
   i. $\int_0^{2\pi} e^{cos\theta} \sin(\theta + sin\theta)\, d\theta = 0$
   ii. $\int_0^{2\pi} e^{cos\theta} \cos(\theta + sin\theta)\, d\theta = 0$
3. Using Cauchy's Integral formula, evaluate $\int_C^{\cdot} \frac{dz}{z^3(z+4)}$, where C is the circle $|z| = 2$.
4. Obtain the expansion of $f(z) = \frac{z^2-1}{(z+2)(z+3)}$, in the powers of z in the region
   (i) $|z| < 2$ (ii) $2 < |z| < 3$ (iii) $|z| > 3$.
5. Prove that $\frac{1}{4z-z^2} = \sum_{n=0}^{\infty} \frac{z^{n-1}}{4^{n+1}}$, where $0 < |z| < 4$.
6. Verify Cauchy's integral theorem for $f(z) = z^2$ around the circle $|z| = 1$.
7. Evaluate $\int_{|z|=2}^{\cdot} \frac{e^{2z}}{(z-1)^4} dz$ using Cauchy's integral formula.
8. Find the expansion of $f(z) = \frac{1}{(z^2+1)(z^2+2)}$ in powers of z, when $|z| < 1$.

Practical-4: Calculus of Residues

1. Find the residue of $f(z) = \frac{z^2+2z}{(z+1)^2(z+4)}$ at its poles.
2. Evaluate $\int_{|z|=3}^{\cdot} \frac{e^z}{z(z-1)^2} dz$ by Cauchy's residue theorem.

3. Evaluate $\int_C \frac{3z^2+2}{(z-1)(z^2+9)} dz$ by Cauchy's residue theorem, where C is (i) The circle $|z-2| = 2$ (ii) The circle $|z| = 4$
4. Use the Contour integration to evaluate $\int_0^{2\pi} \frac{d\theta}{5+3cos\theta}$.
5. Evaluate by Contour integration $\int_{-\infty}^{\infty} \frac{1}{x^4+13x^2+36} dx$.
6. Find the sum of residues of $f(z) = \frac{e^z}{z^2+a^2}$ at its poles.
7. Evaluate $\int_{|z|=2} \frac{dz}{z^3(z+4)}$ by Cauchy's residue theorem.
8. Evaluate $\int_0^{\infty} \frac{x^2 dx}{(x^2+1)(x^2+4)}$ by contour integration.

Practical 5: Theory of ordinary differential equations

1. Show that $f(x,y) = xy^2$ satisfies the Lipschitz condition on the rectangle $R: |x| \leq 1, |y| \leq 1$, but does not satisfy the Lipschitz condition on the strip $S: |x| \leq 1, |y| \leq \infty$.
2. Prove that sin$2x$ and cos$2x$ are solutions of $y'' + 4y = 0$ and these solutions are linearly independent.
3. Prove that $1, x, x^2$ are linearly independent. Hence, form the differential equation whose solutions are 1, x and x^2.
4. Examine whether the set of functions 1, x^2, x^3 are linearly independent or not.
5. Solve by method of variation of parameters $y'' + a^2y = cosec(ax)$
6. Solve by method of variation of parameters $y'' + y - x = 0$
7. Show that functions $1 + x$, $x^2, 1 + 2x$ are linearly independent.
8. Examine whether e^{2x} and e^{3x} are linearly independent solution of differential equation $y'' - 5y' + 6y = 0$ or not?
9. Solve by method of variation of parameters, $y'' + 3y = sec3x$.

Practical 6 –Simultaneous Differential Equations

1. Solve : (i) $\frac{dx}{x^2z} = \frac{dy}{0} = \frac{dz}{-x^2}$ and (ii) $\frac{dx}{\tan x} = \frac{dy}{\tan y} = \frac{dz}{\tan z}$
2. Solve : (i) $\frac{dx}{xy} = \frac{dy}{y^2} = \frac{dz}{xyz-zx^2}$ and (ii) $\frac{dx}{y} = \frac{dy}{x} = \frac{dz}{xyz^2(x^2-y^2)}$
3. Solve : $\frac{dx}{y+z} = \frac{dy}{z+x} = \frac{dz}{x+y}$
4. Solve : $\frac{adx}{yz(b-c)} = \frac{bdy}{zx(c-a)} = \frac{cdz}{xy(a-b)}$
5. Solve : $\frac{dx}{x^2-y^2-z^2} = \frac{dy}{2xy} = \frac{dz}{2xz}$
6. Solve : $\frac{dx}{z(x+y)} = \frac{dy}{z(x-y)} = \frac{dz}{x^2+y^2}$

7. Solve : $\frac{dx}{\sin(x+y)} = \frac{dy}{\cos(x+y)} = \frac{dz}{z}$

8. Solve : $\frac{dx}{z^2} = \frac{ydy}{xz^2} = \frac{dz}{xy}$

9. Solve : $\frac{dx}{y^2(x-y)} = \frac{dy}{-x^2(x-y)} = \frac{dz}{z(x^2+y^2)}$.

Practical - 7 :Total (Pfaffian) Differential Equations

1. Show that the following differential equations are integrable. Hence solve them
   (i) $(y^2 + z^2 + x^2)dx - 2xydy - 2xzdz = 0$
   (ii) $2yzdx + zxdy - xy(1 + z)dz = 0$
2. Solve : $yz^2(x^2 - yz)dx + zx^2(y^2 - xz)dy + xy^2(z^2 - xy)dz = 0$
3. Solve : $\frac{yz}{x^2+y^2}dx - \frac{xz}{x^2+y^2}dy - tan^{-1}\frac{y}{x}dz = 0$
4. Solve : $zydx = zxdy + y^2dz = 0$
5. Solve : $(x^2 - yz)dx + (y^2 - xz)dy + (z^2 - xy)dz = 0$
6. Solve : $(2x^2 + 2xy + xz^2 + 1)dx + dy + 2zdz = 0$
7. Solve : $(y + z)dx + dy + dz = 0$
8. Show that the equation $yz^2(x^2 - yz)dx + zx^2(y^2 - xz)dy + xy^2(z^2 - xy)dz = 0$ is integrable. Is it exact? Verify.

Practical 8(A) : Difference Equations

1. Form the difference equation corresponding to the following general solution:
   (a) $y = c_1x^2 + c_2x + c_3$
   (b) $y = (c_1 + c_2n)(-2)^n$
2. Show that $y_x = c_1 + c_2.2^x - x$ is a solution of difference equation $y_{x+2} - 3y_{x+1} + 2y_x = 1$.
3. Formulate the Fibonnaci difference equation and solve it.
4. Solve the following difference equations:
   (a) $y_{x+1} - 3y_x = 1$
   (b) y_{x+1}-$3y_x = 0,\ \ y_0 = 2$
5. Solve the following non-homogeneous linear difference equations:
   (i) $y_{x+2} - 4y_x = 9x^2$
   (ii) $\Delta y_x + \Delta^2 y_x = sinx$
6. Solve: $y_{x+2} - 4y_{x+1} + 3y_x = 3^x + 1$.
7. Solve: $y_{x+2} - 4y_{x+1} + 4y_x = 3x + 2^x$.
8. Solve: $u_{x+2} - 5u_{x+1} + 6u_x = 36$.

Practical – 8 (B): Numerical differentiation

1. Find the first and second derivatives of the function tabulated below at $x = 1.9$

x	1.0	1.2	1.4	1.6	1.8	2.0
$f(x)$	0	0.128	.544	1.296	2.432	4.0

2. Find first and second derivatives at $x = 0$ from the following table:

x	0	1	2	3	4	5
$f(x)$	4	8	15	7	6	2

3. Find the value of $sec(310)$ from the following table:

x	31	32	33	34
$sec(x)$	0.6008	0.6249	0.6494	0.6745

4. Find first derivative using Stirling's formula at $x = 0.5$:

x	0.35	0.4	0.45	0.5	0.55	0.6	0.65
$f(x)$	1.521	1.506	1.488	1.467	1.444	1.418	1.389

5. Find the maximum value of $f(x)$ from the following table:

x	3	4	5	6	7	8
$f(x)$	0.205	0.240	0.259	0.262	0.250	0.224

6. Find the maximum value of y from the following table:

x	0	2	3	4	7	9
$f(x)$	4	26	58	112	466	922

7. Find the first derivative at $x = 4$ by using Stirling's formula:

x	1	2	3	4	5	6
$f(x)$	1	3	7	13	21	31

8. Find the maximum and minimum values of $f(x)$:

x	0	1	2	3	4	5
$f(x)$	0	0.25	0	2.25	16.0	56.25

Note: Practical problems based on each unit are not limited to the given ones, but any other related challenging and application-oriented problems may also be evaluated in the practical sessions.

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SEC- 2

MTH 404: Vector Calculus (Period: 30 Clock hours)

Course Description:

This is a skill development course of vector algebra and its calculus for S. Y. B.Sc. students

Prerequisite Course(s): Secondary school level knowledge of elementary physics and mathematics.

General Objective: The general objectives are to acquire skills of vectors algebra, vector valued functions, operators like del and curl and line and surface integrals.

Learning Outcomes:

a) understand scalar and vector products
b) understand vector valued functions and their limits and continuity and use them to estimate velocity and acceleration of partials.
c) Calculate the curl and divergence of a vector field.
d) Set up and evaluate line integrals of functions along curves.

Unit -1: Product of Vectors **Marks-15**

1.1 Scalar Product
1.2 Vector Product
1.3 Scalar Triple Product
1.4 Vector Product of Three Vectors
1.5 Reciprocal Vector

Unit-2: Vector functions **Marks-15**

1.1 Vector functions of a single variable.
1.2 Limits and continuity.
1.3 Differentiability, Algebra of differentiation.
1.4 Curves in space, Velocity and acceleration.
1.5 Vector function of two or three variables.
1.6 Limits, Continuity, Partial Differentiation

Unit-3: The Vector Operator Del **Marks-15**

2.1 The vector differentiation operator del.
2.2 Gradient.
2.3 Divergence and curl.
2.4 Formulae involving del. Invariance.

Unit-4: Vector Integration **Marks-15**

3.1 Ordinary integrals of vectors.
3.2 Line integrals.
3.3 Surface integrals.

Recommended Book:

1. Vector Analysis by Murray R Spiegel, Schaum’s Series, McGraw Hill Book Company.

Reference Book:

1. Vector Calculus by Shanti Narayan and P.K. Mittal, S. Chand & Co., New Delhi

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NORTH MAHARASHTRA UNIVERSITY, JALGAON

Equivalence courses for S. Y. B. Sc. (Mathematics)

Effective from 2019

Semester	Old course (June 2016)	New course (June 2019)
Sem-I	MTH 231 : Calculus of Several Variables	MTH 301 : Calculus of Several Variables
	MTH 232(A): Algebra	MTH 302(A) : Group Theory
	MTH 232(B): Theory of Groups	MTH -302(B): Theory of Groups and Codes
	MTH 233 : Practical Course based on MTH-232 & MTH-232	MTH 303 : Practical paper based on MTH 301 & MTH 302
Sem-II	MTH 241 : Complex Variables	MTH 401 : Complex Variables
	MTH 242(A): Differential Equations	MTH 402 : Differential Equations
	MTH 242(B): Differential and Difference Equations	MTH-402 (B): Differential Equations and Numerical Methods
	MTH 243 : Practical Course based on MTH-241 & MTH-242	MTH 403 : Practical paper based on MTH 401 & MTH 402